1. (10 points) Find all critical numbers of the function

$$f(x) = \frac{x^2}{3 - x}$$

2. (10 points) Find the limits.

a) $\lim_{x \to -\infty} \dfrac{5x^3 - 3x + 1}{2x^3 + x^2}$

b) $\lim_{x \to -\infty} \dfrac{\sqrt{3x^2 - 1}}{2x - 3}$

3. (10 points) Given the derivative $f'(x)$ of the function $f(x)$, list all intervals on which $f(x)$ is increasing.

$$f'(x) = \frac{(2x-1)^2(x-4)}{x^3(x^2+3)}$$

4. (15 points) Find the absolute maximum and the absolute minimum values on the closed interval $[0, \frac{3\pi}{2}]$ of the function

$$f(x) = \sin x + \cos^2 x$$

5. (15 points) An observer is positioned 3 km away from a rocket launch pad. How fast is the distance between the rocket and the observer increasing, when the rocket is 4 km above the ground and is moving straight up at the speed of 300 m/sec?

6. (15 points) Water is leaking from a conical cup at the constant rate of 2 cm^3/min. The height of the cup is 12 cm and the radius of the top is 4 cm. How fast is the level of the water in the cup decreasing when the water is 3 cm deep? (The volume of a right circular cone is given by the formula $V = \frac{1}{3}\pi r^2 h$.)

7

10. (35 points) Let $g(x) = \frac{x^2}{3x-2}$. To save you time, I'm giving you the derivatives of g: $g'(x) = \frac{3x^2-4x}{(3x-2)^2}$ and $g''(x) = \frac{8}{(3x-2)^3}$.

a. Give the vertical asymptotes. (If there are none, say so.) Remember to justify your answer.

b. Give the horizontal asymptotes. (If there are none, say so.) Remember to justify your answer.

c. Give the intervals of increasing and decreasing, and give all local maxima and local minima.

d. Give the intervals of concavity and the inflection points.

e. Sketch the graph of g.