2022 Zassenhaus Group Theory and Friends Conference
Hofstra University, Long Island NY
May 27–29, 2022 (Online)
and June 4, 2022 (in person)

PROGRAM & ABSTRACTS

Organizing Committee

Zoran Sunic, Hofstra University, Long Island, New York
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Arturo Magidin, University of Louisiana at Lafayette, Lafayette, Louisiana
Hung Tong-Viet, Binghamton University, Binghamton, New York

2022 Zassenhaus Group Theory and Friends Conference
May 27–29 (online) and June 4 (in person)

Conference Program

Online Timetable

Friday, May 27, Midday Session

TIME (EDT)

12:00–12:30 Agota Figula (University of Debrecen)
Akivis algebra of abelian extensions and tangent prolongations of loops

12:35–13:05 Chris Schroeder (Binghamton University)
The 2-partsof relative character degrees

13:10–13:40 Wil Cocke (Augusta University)
A groupth theoretic correspondence

13:45–14:15 Jeffrey Riedl (University of Akron)
Iterated commutators of powers of an element

Friday, May 27, Afternoon Session

TIME (EDT)

15:45–16:15 Ryan McCulloch (Elmira College)
Central products and the Chermak-Delgado lattice

16:20–16:50 Mark Sapir (Vanderbilt University)
A survey about F

16:55–17:25 Santiago Barrera Acevedo (Monash University)
Cocyclic two-circulant core Hadamard matrices

17:30–18:00 Heiko Dietrich (Monash University)
Galois trees for p-group of maximal class

Saturday May 28, Morning session

TIME (EDT)

8:40–9:10 Viji Thomas (Indian Institute of Science Education and Research Thiruvananthapuram)
Schur's Exponent Conjecture and related problems

9:15–9:45 Patali Komma (Indian Institute of Science Edutation and Research Thiruvananthapuram)
Non-inner automorphisms of order p in finite p-groups of coclass 4 and 5

9:50–10:20 Marshal Sampson (Akwa Ibom State University, Ikot Akpaden, Nigeria)
On existence of minimal generating sets and maximal independent sets in groups and the additive semigroup of integers

10:25–10:55 Constantino Delizia (University of Salerno)
On anticommutative Engel groups

Saturday May 28, Midday Session

TIME (EDT)

12:00–12:30 Juan Martinez Madrid (Universitat de Valencia)
On the fields of values of irreducible charcters

12:35–13:05 Mihai-Silviu Lazorec (Alexandru Ioan Cuza University of Iasi, Romania)
A generalization of a result on the sum of element orders of a finite group

13:10–13:40 Chimere Anabanti (University of Pretoria)
Which finite groups are filled?

13:45–14:15 Tung Nguyen (Western University)
Zeta functions of the joint algebras over finite fields

Saturday May 28, Afternoon Session

TIME (EDT)

15:10–15:40 Lucas Gagnon (University of Colorado Boulder)
A Hopf algebra approach to induced supercharacters

15:45–16:15 Jonathan Doane (Binghamton University)
Subquasivariety lattice of Boolean semirings

16:20–16:50 Mark Lewis (Kent State University)
Groups having all elements off a normal subgroup with prime power order

16:55–17:25 Zelriya (Yalcin) Karatas (University of Cincinnati Blue Ash College)
A brief history on classification of groups whose proper subgroups satisfy certain conditions

Sunday May 29, Morning Session

TIME (EDT)

8:40–9:10 Tony Nixon Mavely (IISER Thiruvananthapuram)
Bounds on the size of Schur multiplier of p-groups with graph theory

9:15–9:45 Ekaterina Kompantseva (Financial University, Moscow)
and Thi Quynh Trang Nguyen (FPT University)
Filial rings on quotient divisible abelian groups

9:50–10:20 Gareth Jones (University of Southampton)
Simple groups of order a product of six primes

10:25–10:55 J. Miquel Martínez (Universitat de Valencia)
The number of irreducible characters in a Brauer block

In Person Timetable

Saturday June 4, Morning Session

TIME (EDT)	
10:30–10:55	Arturo Magidin (University of Louisiana at Lafayette) **Generalizing the Chermak-Delgado measure**
11:05–11:30	Carmine Monetta (University of Salerno) **On the solubilizer of an element in finite groups**
11:30–12:00	Coffee Break
12:00–12:25	David Burrell (University of Florida) **The groups of order** 1024**: a new total**
12:35–13:00	Justin Lynd (University of Louisiana at Lafayette) **Punctured groups for exotic fusion systems**
13:00–14:30	Lunch Break

Saturday June 4, Afternoon Session

TIME (EDT)	
14:30–14:55	Zoran Sunic (Hofstra University) **Frobenius groups leading to branch groups**
15:05–15:30	Tuval Foguel (Adelphi University) **Semi-partition of a group**
15:40–16:05	Andrew Velasquez-Berroteran (Adelphi University) **Equal coverings of finite groups**
16:05–16:30	Coffee Break
16:30–16:55	Casey Donoven (Montana State University Northern) **Synchronizing automata for Thompson's Group** V
15:05–15:30	Luise Kappe (Binghamton University) **On the nonabelian tensor product of cyclic groups of** p**-power order,** p **an odd prime**
18:30	**Conference Dinner**

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Abstracts

In order of presentation

Online Sessions

Akivis algebra of abelian extensions and tangent prolongations of loops

Agota Figula, University of Debrecen, Institute of Mathematics

The tangent Akivis algebra of a smooth loop is the tangential object determined by the third-order taylor polynomial of the multiplication function of the loop. It is endowed with a bilinear antisymmetric and a trilinear operation defined by the infinitesimal commutator and associator of the loop satisfying the so-called Akivis identity. The aim of our work is to study extensions of Akivis algebras in the framework of binary-ternary algebras and to investigate tangent Akivis algebras of smooth loops, which are abelian extensions of loops. We apply the obtained results to the determination of the tangent Akivis algebras of the tangent prolongation of differentiable loops, which forms a subclass of abelian extensions.

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The 2-parts of relative character degrees

Chris Schroeder, Binghamton University

Let G be a finite group. The following result, proved by Higgs and Moretó, was used by Navarro and Tiep in their proof of Brauer's height zero conjecture for 2-blocks of maximal defect: suppose Z is a normal subgroup of G and θ is a G-invariant irreducible character of Z. If the relative degree $\chi(1)/\theta(1)$ is odd for every irreducible character χ of G lying over θ, then G/Z is solvable. The original proof of this result used the language of projective representations. We give a character theoretic proof and report progress on a conjecture extending this result to the case where the relative degrees are not divisible by 4. Our work extends to the projective setting a result of Lewis that if G is nonsolvable and $4 \nmid \chi(1)$ for all irreducible characters χ of G, then $G \cong A_7 \times S$ with S solvable.

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A groupth theoretic correspondence

Wil Cocke, Augusta University

Graph theory should show up early and often in an undergraduate math and computer science curriculum. Recent work in model repair has focused on using the symmetry of a model to reduce the search space for needed repairs. I will summarize some of this work and present a correspondence theorem between symmetric subgraphs of the original graphs and subgraphs of the quotient. This correspondence, like the correspondence theorem in group theory creates a bijection of posets in the manner of Galois theory, but is readily accessible to most students.

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Iterated commutators of powers of an element

Jeffrey Riedl, University of Akron

Let x, y be elements in a group G. Let $d(k)$ denote the iterated commutator element $[\ldots[[x,y],y],\ldots,y]$ where y appears k times. This defines a sequence $\mathcal{D} = \{d(0), d(1), d(2), \ldots\}$. Let m be a positive integer. We present a formula and some techniques for expressing the element $[\ldots[[x,y^m],y^m],\ldots,y^m]$ as a product of elements of $\mathcal{D}$ in a convenient way. We apply these techniques to gain information about how certain normal subgroups of the regular wreath product $W = \mathbb{Z}_{p^2} \wr \mathbb{Z}_{p^2}$ are permuted in a natural action of the group of units $U(p^2)$ via automorphisms on W.

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Central products and the Chermak-Delgado lattice

Ryan McCulloch, Elmira College

The Chermak-Delgado lattice of a finite group G is a modular, self-dual sublattice of the lattice of subgroups of G. In this talk we consider the Chermak-Delgado lattice of a central product. This is joint research with William Cocke.

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A survey about F

Mark Sapir, Vanderbilt University

I am going to talk about applications of the Thompson group F to links and knots discovered by Vaughan Jones. Joint work with Gili Golan-Polak.

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Cocyclic two-circulant core Hadamard matrices

Santiago Barrera Acevedo

The two-circulant core (TCC) construction for Hadamard matrices (HM) uses two sequences with almost perfect autocorrelation to construct a HM. A research problem of K. Horadam asks whether such matrices are cocyclic. Using ideas from permutation groups, we prove that the order of a cocyclic TCC HM coincides with the order of a HM of Paley type, Sylvester type or certain multiples of these orders. In addition, we show that there exist cocyclic TCC HMs at all allowable order less or equal to 1000 with at most one exception.

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Galois trees for p-groups of maximal class

Heiko Dietrich, Monash University

Leedham-Green & Newman defined the *coclass* of a finite p-group of order p^n and nilpotency class c as $r = n - c$. The investigation of the p-groups of a fixed coclass led to deep results in p-group theory (see the book of Leedham-Green & McKay), to applications (e.g. the investigation of Schur multipliers or automorphism groups of p-groups), and to generalisations to other algebraic objects (e.g. algebras or semigroups). In the last decade, the focus in coclass theory is on the investigation of the *coclass graph* $\mathcal{G}(p, r)$ associated with the finite p-groups of coclass r: the vertices of this graph are (isomorphism type representatives of) the finite p-groups of coclass r, and there is an edge between two groups G and H if and only if G is isomorphic to $H/\gamma(H)$ where $\gamma(H)$ is the last non-trivial term in the lower central series of H. The graph $\mathcal{G}(p, r)$ is infinite and rich in structure. Indeed, it is a central conjecture that it can be described completely by a finite subgraph and several "periodicity patterns". The aim of this talk is to describe some recent new periodicity results for $\mathcal{G}(p, 1)$ and discuss some open problems. This is joint work with Alexander Cant, Bettina Eick, and Tobias Moede.

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Schur's Exponent Conjecture and related problems

Viji Thomas, Indian Institute of Science Education
and Research Thiruvananthapuram

We will discuss Schurs exponent conjecture, and discuss the progress made towards the conjecture in recent years. Finally we will describe our contribution towards this conjecture. Time permitting we will also discuss some problems in graph theory and apply it to obtain the size of the Schur Multiplier for some classes of groups.

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Non-inner automorphisms of order p in finite p-groups of coclass 4 and 5

PATALI KOMMA, Indian Institute of Science Education and Research Thiruvananthapuram

A long-standing conjecture asserts that every finite nonabelian p-group admits a non-inner automorphism of order p. In this talk first we will give criteria for the existence of a derivation $\delta : G \to A$ such that $\delta(\gamma_2(G)) \neq 1$ when G is an extra-special group of exponent p. Moreover, we show that every finite p-group G that is not powerful has a normal subgroup N for which $G/N = U\ G/N = U \times V$ where $U\ U \leq Z(G/N)$ is elementary abelian and V is an extra-special group of exponent p. This allows us to construct an automorphism of order p not fixing the commutator subgroup in finite p-groups with a cyclic center. As a consequence, we deduce that finite p-groups of coclass 4 and 5 admit a non-inner automorphism of order p ($p \geq 5$) We also prove the conjecture for an odd order nonabelian p-group G with cyclic center satisfying $C_G(G^p\gamma_3(G)) \cap Z_3(G) \leq Z(\Phi(G))$.

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On existence of minimal generating sets and maximal independent sets in groups and the additive semigroup of integers

MARSHAL SAMPSON, Akwa Ibom State University, Ikot, Akpaden, Nigeria.

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On anticommutative Engel groups

Constantino Delizia, University of Salerno

In this talk, a group is called anticommutative if two of its elements commute only when they are powers of the same element. This is equivalent to require that every abelian subgroup is locally cyclic. Finite anticommutative groups were classified (by Zassenhaus in the soluble case, and by Suzuki in the insoluble case). We deal with infinite anticommutative groups, and completely describe their structure in a large class of generalized soluble groups (namely, the locally virtually soluble groups). Furthermore, we focus on anticommutative Engel groups, and we prove that they are locally nilpotent. Finally, we show that anticommutative n-Engel groups are nilpotent of class at most n. This is a joint work with Chiara Nicotera.

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On the fields of values of irreducible characters

Juan Martínez Madrid, Universitat de Valencia

There are many results that relate the structure of a group with the fields of values of its irreducible characters. We will begin by reviewing some classical results. Then, we will focus on studying the maximum number of times that an extension of the rational numbers appears as the field of values of an irreducible character.

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A generalization of a result on the sum of element orders of a finite group

Mihai-Silviu Lazorec, Alexandru Ioan Cuza University of Iasi, Romania

Let G be a group of order n and H be a subgroup of order m of G. Denote by $\psi_H(G)$ the sum of element orders relative to H of G. It is known that if G is nilpotent, then $\psi_H(G) \leq \psi_{H_m}(G)$, where H_m is the unique subgroup of order m of C_n. We show that this inequality does not hold for infinitely many finite solvable groups.

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Which finite groups are filled?

Chimere Anabanti, University of Pretoria

A non-empty subset S of a group G is called a product-free set if S and SS have no element in common. Let S be a maximal by inclusion product-free set in a finite group G. We say that S fills G if every non-identity element of G is contained in the union of S and SS. A finite group G is called a filled group if every maximal by inclusion product-free set in G fills G. In this talk, we shall discuss the known finite filled groups.

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Zeta functions of the joint algebras over finite fields

Tung Nguyen, Western University

A natural model of a network of oscillators is a G-circulant matrix where G is a (cyclic) group. We observe that the set of all G-circulant matrices is naturally isomorphic to the group algebra k[G] of G. Consequently, the Artin-Wedderburn theorem for k[G] allows us to describe the eigenspectrum of these circulant networks concretely. Due to this explicit description, many problems involving circulant matrices have closed-form or analytical solutions. In our investigation of the non-linear dynamics on multilayer networks of oscillators, the joins of circulant graphs appear quite often, and they provide new exciting phenomena. This naturally leads us to investigate the joint algebra, an algebra obtained by joining several group algebras. We will introduce this joint algebra, some of its properties, and then describe its zeta function when the base field is a finite field. This is joint work with Sunil Chebolu, Jon Merzel, Jan Minac, Lyle Muller, Federico Pasini, and Nguyen Duy Tan.

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A Hopf algebra approach to induced supercharacters

Lucas Gagnon, University of Colorado, Boulder

This talk is about a new way to compute the induction functor from the (complex) representations of the finite unipotent upper triangular group $\mathrm{UT}_n(\mathbb{F}_q)$ to the representations of the finite general linear group $\mathrm{GL}_n(\mathbb{F}_q)$, for some finite field $\mathbb{F}_q$. A key tool in the study of $\mathrm{GL}_n(\mathbb{F}_q)$ is a Hopf algebra structure (due to Zelevinsky) on the representations of these groups: this lets us build up ("multiply") and break down ("comultiply") representations is a systemic manner. My approach will be to build an analogous Hopf structure on the representations of $\mathrm{UT}_n(\mathbb{F}_q)$ for which induction is a Hopf algebra homomorphism; the upshot is that Frobenius reciprocity and a theorem of Aguiar–Bergeron–Sottile makes computing the induction map equivalent to a collection of iterated comultiplications in this novel Hopf algebra. For certain nice characters arising from the study of supercharacter theory, this becomes a previously understood combinatorial problem and the resulting $\mathrm{GL}_n(\mathbb{F}_q)$-characters can be described in terms of graph colorings.

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Subquasivariety lattice of Boolean semirings

Jonathan Doane, Binghamton University

This talk presents the findings of dissertation *Boolean twice monoids*. We characterize the subquasivariety lattice of Boolean semirings via a countably infinite set of algebras (with cardinalities given by sequential Fibonacci numbers).

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Groups having all elements off a normal subgroup with prime power order

Mark Lewis, Kent State University

We consider a finite group G with a normal subgroup N so that all elements of $G \setminus N$ have prime power order. We prove that if there is a prime p so that all the elements in $G \setminus N$ have p-power order, then either G is a p-group or $G = PN$ where P is a Sylow p-subgroup and $(G, P, P \cap N)$ is a Frobenius-Wielandt triple. We also prove that if all the elements of $G \setminus N$ have prime power orders and the orders are divisible by two primes p and q, then G is a $\{p, q\}$-group and G/N is either a Frobenius group or a 2-Frobenius group. This builds on the classification by Higman of solvable groups where all elements have prime power order. If all the elements of $G \setminus N$ have prime power orders and the orders are divisible by at least three primes, then all elements of G have prime power order and G/N is nonsolvable. The nonsolvable groups where all elements have prime power order have been classified by Brandl extending the work of Suzuki who determined the simple groups with all elements having prime power order.

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A brief history on classification of groups whose proper subgroups satisfy certain conditions

Zekeriya (Yalcin) Karatas, University of Cincinnati Blue Ash College

One of the most active areas in group theory is the classification of groups whose proper subgroups satisfy certain conditions. Many famous mathematicians studied these types of problems including Dedekind and Baer. The conditions used for the subgroups became more generalized and increased by time, and as a result, these types of problems became more attractive and challenging. In this talk, the history of these types of problems including the most significant results will be presented. Recent studies in this area will be presented as well.

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Bounds on the size of Schur multiplier of p-groups with graph theory

Tony Nixon Mavely, IISER Thiruvananthapuram

The Schur multiplier of a given group has been of considerable interest to mathematicians, and thus determining its size is a problem that has generated significant interest. For special groups of rank k, the bound on the size of the Schur multiplier was given for two extreme values of k. We provide a bound for the size of the Schur multiplier for all special groups of rank between the given extremes using a bound on the number of triangles for a given graph. We also improve the upper bound for the size of the Schur multiplier of groups with nilpotency class greater than or equal to 3. As a corollary, we also improve the bounds for the size of the Schur multiplier for groups of given coclass r.

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Filial rings on quotient divisible Abelian groups

Ekaterina Kompantseva, Financial University, Moscow
and Thi Quynh Trang Nguyen, FPT University

A ring on an Abelian group G is a ring, whose additive group is isomorphic to G.

L. Fuchs fomulated the problem of describing Abelian groups that admits regular ring structure. Filial rings are a generalization of regular rings.

In the present paper, additive groups of regular rings and groups on which any associative ring is filial are described in the class of quotient divisible Abelian groups of rank 1.

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Simple groups of order a product of six primes

Gareth Jones, University of Southampton

In the 1890s Burnside, Frobenius and Hölder classified the nonabelian simple groups whose orders are products of at most five primes, namely $\mathrm{PSL}_2(p)$ for $p = 5, 7, 11$ and 13. What about products of six primes? By the classification of finite simple groups, the only possibilities are $\mathrm{PSL}_2(8)$, $\mathrm{PSL}_2(9)$ and $\mathrm{PSL}_2(p)$ for certain primes p such as 23 and 29. Peter Neumann asked whether there are infinitely many such primes. Joint work with Sasha Zvonkin (Bordeaux) uses the Bateman–Horn Conjecture from Number Theory, supported by extensive computer searches, to give strong evidence that there are infinitely many of them.

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The number of irreducible characters in a Brauer block

J. Miquel Martínez, Universitat de Valencia

Let G be a finite group, p a prime and B a Brauer p-block with defect group D. Exploring properties of D through the knowledge of some invariants of B is a classical problem in character theory. Let $k(B)$ denote the number of irreducible characters in B. We discuss certain results concerning the possible structures of D for small values of $k(B)$. This is joint work with Noelia Rizo and Lucia Sanus.

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In Person Sessions

Generalizing the Chermak-Delgado measure

Arturo Magidin, University of Louisiana at Lafayette

Let G be a finite group, H a subgroup. The Chermak-Delgado measure of H in G is $m(H) = |H|\,|C_G(H)|$. The subgroups with maximum measure form a sublattice of the lattice of subgroups of G with many nice properties.

In joint work with Elizabeth Wilcox, Luise-Charlotte Kappe, and William Cocke, we are looking at generalizations that involve marginal subgroups. I will discuss the idea, some examples, the obstacles that arise to a direct generalization, and some of the ways in which they can be overcome.

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On the solubilizer of an element in finite groups

Carmine Monetta, University of Salerno

The solubility graph associated with a finite group G is a simple graph whose vertices are the elements of G, and there is an edge between two distinct vertices if and only if they generate a soluble subgroup.

The aim of this talk is to present some new results related to the solubility graph, underlining how graph properties affect the structure of the group G. In particular, we will focus on the set of neighbors of a vertex x, which we call the *solubilizer* of x in G, investigating both arithmetic and structural properties of this set.

This is a joint work with Banafsheh Akbari and Costantino Delizia.

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The groups of order 1024: a new total

David Burrell, University of Florida

The problem of finding all the groups of a fixed order n is an old one, initiated by an 1878 paper of Cayley. Significant progress has been made for small n ($n \leq 20,000$) since his call to action. In this presentation we will discuss a recent enumeration of the groups of order 2^{10} which shows that there are 49487367289 such groups as opposed to 49487365422 isomorphism classes reported just over 20 years ago. Topics include the p-group generation algorithm and solving large computational problems in group theory via parallelization within a High-performance computing environment.

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Punctured groups for exotic fusion systems

Justin Lynd, University of Louisiana at Lafayette

The fusion system of a finite group G at a prime p is a category whose objects are the subgroups of a fixed Sylow p-subgroup S, and where the morphisms are the conjugation homomorphisms induced by the elements of G. The notion of a saturated fusion system is abstracted from this standard example and leads to exotic examples, namely fusion systems not realizable by a finite group in the above fashion. Given an exotic fusion system, one might want to ask: how close is it to being realizable by a finite group? This leads to the study of punctured groups, which are the most group-like objects that one can hope to associate with an exotic fusion system. It turns out that some exotic systems have them, while some do not. I plan to give background on these topics, look at some examples such as the Benson-Solomon exotic fusion systems at the prime 2, and mention an application of the existence of a punctured group to the topology of classifying spaces. This is joint work with Assaf Libman and Ellen Henke.

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Frobenius groups leading to branch groups

Zoran Sunic, Hofstra University

A Frobenius group is a finite, transitive, permutation group in which no nontrivial group element fixes more than one point, but some nontrivial elements do fix a point (thus, the action is not free). Starting with various Frobenius groups we construct families of groups of tree automorphisms with remarkable properties. The previous construction (by Alejandra Garrido and the author) of the first family of regular branch groups with infinite rigid kernels can be understood as an instance of this approach, as can some examples constructed by Rachel Skipper. The best known example that fits in this general construction is the Hanoi Towers group. The understanding of these older examples through the lens of Frobenius groups yields new examples and results.

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Semi-partition of a group

Tuval Foguel, Adelphi University

A group partition is a group cover in which the components have trivial pairwise intersection. Based on an idea of I. M. Isaacs in this talk

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Equal coverings of finite groups

Andrew Velasquez-Berroteran, Adelphi University

A covering of a group is collection of proper subgroups whose set-theoretic union is the group. An equal covering is a covering where all proper subgroups are of the same order. We will look at some theorems and a look at GAP code relevant to finding which sort of finite groups have an equal coverings and which do not. Some general history of the research done in coverings within the past century will also be mentioned.

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Synchronizing Automata and Thompson's Group V

Casey Donoven, Montana State University Northern

Thompson's group V is a group of homeomorphisms of Cantor space that can be defined in many different ways, including as a group of automata acting on infinite strings over a two-letter alphabet. It was shown in Bleak et al. that automorphisms of V are induced by automata that are synchronizing and whose inverse is synchronizing. Relaxing this condition produces isomorphisms of V to subgroups or supergroups of itself. In this talk, I will describe synchronizing automata and the subgroups of V they produce.

Joint work with Feyishayo Olukoya, Collin Bleak, and Julius Jonašus.

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On the nonabelian tensor product of cyclic groups of p-power order, p an odd prime

Luise Kappe, Binghamton University

The nonabelian tensor product of two groups G and H was introduced by R. Brown and J.-L. Loday in connection with applications in homotopy theory of a generalized Van Kampen theorem. The nonabelian tensor product $G \otimes H$ is the group generated by symbols $g \otimes h$ with relations $gg' \otimes h = ({}^{g}g' \otimes {}^{g}h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^{h}g \otimes {}^{h}h')$, where G and H act on each other via automorphisms in a compatible way and on themselves via conjugation. Good progress has been made in the computation of large classes of groups in the case of the nonabelian tensor square $G \otimes G$ where the actions are conjugation, which are always compatible.

We show that the nonabelian tensor product of two cyclic groups is an abelian group of at most rank 2. In case G and H are cyclic groups of p-power order, p an odd prime, we classify all compatible actions and show that $G \otimes H$ is cyclic with $|G \otimes H| = \min(|G|, |H|)$. In case G and H are cyclic 2-groups we give examples such that $G \otimes H$ is an abelian group of rank 2.

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Abstracts

Alphabetical by Speaker

Which finite groups are filled?

Chimere Anabanti, University of Pretoria

A non-empty subset S of a group G is called a product-free set if S and SS have no element in common. Let S be a maximal by inclusion product-free set in a finite group G. We say that S fills G if every non-identity element of G is contained in the union of S and SS. A finite group G is called a filled group if every maximal by inclusion product-free set in G fills G. In this talk, we shall discuss the known finite filled groups.

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Cocyclic two-circulant core Hadamard matrices

Santiago Barrera Acevedo

The two-circulant core (TCC) construction for Hadamard matrices (HM) uses two sequences with almost perfect autocorrelation to construct a HM. A research problem of K. Horadam asks whether such matrices are cocyclic. Using ideas from permutation groups, we prove that the order of a cocyclic TCC HM coincides with the order of a HM of Paley type, Sylvester type or certain multiples of these orders. In addition, we show that there exist cocyclic TCC HMs at all allowable order less or equal to 1000 with at most one exception.

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The groups of order 1024: a new total

David Burrell, University of Florida

The problem of finding all the groups of a fixed order n is an old one, initiated by an 1878 paper of Cayley. Significant progress has been made for small n ($n \leq 20,000$) since his call to action. In this presentation we will discuss a recent enumeration of the groups of order 2^{10} which shows that there are 49487367289 such groups as opposed to 49487365422 isomorphism classes reported just over 20 years ago. Topics include the p-group generation algorithm and solving large computational problems in group theory via parallelization within a High-performance computing environment.

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A groupth theoretic correspondence

Wil Cocke, Augusta University

Graph theory should show up early and often in an undergraduate math and computer science curriculum. Recent work in model repair has focused on using the symmetry of a model to reduce the search space for needed repairs. I will summarize some of this work and present a correspondence theorem between symmetric subgraphs of the original graphs and subgraphs of the quotient. This correspondence, like the correspondence theorem in group theory creates a bijection of posets in the manner of Galois theory, but is readily accessible to most students.

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On anticommutative Engel groups

Constantino Delizia, University of Salerno

In this talk, a group is called anticommutative if two of its elements commute only when they are powers of the same element. This is equivalent to require that every abelian subgroup is locally cyclic. Finite anticommutative groups were classified (by Zassenhaus in the soluble case, and by Suzuki in the insoluble case). We deal with infinite anticommutative groups, and completely describe their structure in a large class of generalized soluble groups (namely, the locally virtually soluble groups). Furthermore, we focus on anticommutative Engel groups, and we prove that they are locally nilpotent. Finally, we show that anticommutative n-Engel groups are nilpotent of class at most n. This is a joint work with Chiara Nicotera.

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Galois trees for p-groups of maximal class

Heiko Dietrich, Monash University

Leedham-Green & Newman defined the *coclass* of a finite p-group of order p^n and nilpotency class c as $r = n - c$. The investigation of the p-groups of a fixed coclass led to deep results in p-group theory (see the book of Leedham-Green & McKay), to applications (e.g. the investigation of Schur multipliers or automorphism groups of p-groups), and to generalisations to other algebraic objects (e.g. algebras or semigroups). In the last decade, the focus in coclass theory is on the investigation of the *coclass graph* $\mathcal{G}(p, r)$ associated with the finite p-groups of coclass r: the vertices of this graph are (isomorphism type representatives of) the finite p-groups of coclass r, and there is an edge between two groups G and H if and only if G is isomorphic to $H/\gamma(H)$ where $\gamma(H)$ is the last non-trivial term in the lower central series of H. The graph $\mathcal{G}(p, r)$ is infinite and rich in structure. Indeed, it is a central conjecture that it can be described completely by a finite subgraph and several "periodicity patterns". The aim of this talk is to describe some recent new periodicity results for $\mathcal{G}(p, 1)$ and discuss some open problems. This is joint work with Alexander Cant, Bettina Eick, and Tobias Moede.

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Subquasivariety lattice of Boolean semirings

Jonathan Doane, Binghamton University

This talk presents the findings of dissertation *Boolean twice monoids*. We characterize the subquasivariety lattice of Boolean semirings via a countably infinite set of algebras (with cardinalities given by sequential Fibonacci numbers).

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Synchronizing Automata and Thompson's Group V

Casey Donoven, Montana State University Northern

Thompson's group V is a group of homeomorphisms of Cantor space that can be defined in many different ways, including as a group of automata acting on infinite strings over a two-letter alphabet. It was shown in Bleak et al. that automorphisms of V are induced by automata that are synchronizing and whose inverse is synchronizing. Relaxing this condition produces isomorphisms of V to subgroups or supergroups of itself. In this talk, I will describe synchronizing automata and the subgroups of V they produce.

Joint work with Feyishayo Olukoya, Collin Bleak, and Julius Jonašus.

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Akivis algebra of abelian extensions and tangent prolongations of loops

Agota Figula, University of Debrecen, Institute of Mathematics

The tangent Akivis algebra of a smooth loop is the tangential object determined by the third-order taylor polynomial of the multiplication function of the loop. It is endowed with a bilinear antisymmetric and a trilinear operation defined by the infinitesimal commutator and associator of the loop satisfying the so-called Akivis identity. The aim of our work is to study extensions of Akivis algebras in the framework of binary-ternary algebras and to investigate tangent Akivis algebras of smooth loops, which are abelian extensions of loops. We apply the obtained results to the determination of the tangent Akivis algebras of the tangent prolongation of differentiable loops, which forms a subclass of abelian extensions.

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Semi-partition of a group

Tuval Foguel, Adelphi University

A group partition is a group cover in which the components have trivial pairwise intersection. Based on an idea of I. M. Isaacs in this talk

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A Hopf algebra approach to induced supercharacters

Lucas Gagnon, University of Colorado, Boulder

This talk is about a new way to compute the induction functor from the (complex) representations of the finite unipotent upper triangular group $\mathrm{UT}_n(\mathbb{F}_q)$ to the representations of the finite general linear group $\mathrm{GL}_n(\mathbb{F}_q)$, for some finite field $\mathbb{F}_q$. A key tool in the study of $\mathrm{GL}_n(\mathbb{F}_q)$ is a Hopf algebra structure (due to Zelevinsky) on the representations of these groups: this lets us build up ("multiply") and break down ("comultiply") representations is a systemic manner. My approach will be to build an analogous Hopf structure on the representations of $\mathrm{UT}_n(\mathbb{F}_q)$ for which induction is a Hopf algebra homomorphism; the upshot is that Frobenius reciprocity and a theorem of Aguiar–Bergeron–Sottile makes computing the induction map equivalent to a collection of iterated comultiplications in this novel Hopf algebra. For certain nice characters arising from the study of supercharacter theory, this becomes a previously understood combinatorial problem and the resulting $\mathrm{GL}_n(\mathbb{F}_q)$-characters can be described in terms of graph colorings.

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Simple groups of order a product of six primes

Gareth Jones, University of Southampton

In the 1890s Burnside, Frobenius and Hölder classified the nonabelian simple groups whose orders are products of at most five primes, namely $\mathrm{PSL}_2(p)$ for $p = 5, 7, 11$ and 13. What about products of six primes? By the classification of finite simple groups, the only possibilities are $\mathrm{PSL}_2(8)$, $\mathrm{PSL}_2(9)$ and $\mathrm{PSL}_2(p)$ for certain primes p such as 23 and 29. Peter Neumann asked whether there are infinitely many such primes. Joint work with Sasha Zvonkin (Bordeaux) uses the Bateman–Horn Conjecture from Number Theory, supported by extensive computer searches, to give strong evidence that there are infinitely many of them.

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On the nonabelian tensor product of cyclic groups of p-power order, p an odd prime

Luise Kappe, Binghamton University

The nonabelian tensor product of two groups G and H was introduced by R. Brown and J.-L. Loday in connection with applications in homotopy theory of a generalized Van Kampen theorem. The nonabelian tensor product $G \otimes H$ is the group generated by symbols $g \otimes h$ with relations $gg' \otimes h = ({}^g g' \otimes {}^g h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$, where G and H act on each other via automorphisms in a compatible way and on themselves via conjugation. Good progress has been made in the computation of large classes of groups in the case of the nonabelian tensor square $G \otimes G$ where the actions are conjugation, which are always compatible.

We show that the nonabelian tensor product of two cyclic groups is an abelian group of at most rank 2. In case G and H are cyclic groups of p-power order, p an odd prime, we classify all compatible actions and show that $G \otimes H$ is cyclic with $|G \otimes H| = \min(|G|, |H|)$. In case G and H are cyclic 2-groups we give examples such that $G \otimes H$ is an abelian group of rank 2.

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A brief history on classification of groups whose proper subgroups satisfy certain conditions

Zekeriya (Yalcin) Karatas, University of Cincinnati Blue Ash College

One of the most active areas in group theory is the classification of groups whose proper subgroups satisfy certain conditions. Many famous mathematicians studied these types of problems including Dedekind and Baer. The conditions used for the subgroups became more generalized and increased by time, and as a result, these types of problems became more attractive and challenging. In this talk, the history of these types of problems including the most significant results will be presented. Recent studies in this area will be presented as well.

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Non-inner automorphisms of order p in finite p-groups of coclass 4 and 5

Patali Komma, Indian Institute of Science Education and Research Thiruvananthapuram

A long-standing conjecture asserts that every finite nonabelian p-group admits a non-inner automorphism of order p. In this talk first we will give criteria for the existence of a derivation $\delta : G \to A$ such that $\delta(\gamma_2(G)) \neq 1$ when G is an extra-special group of exponent p. Moreover, we show that every finite p-group G that is not powerful has a normal subgroup N for which $G/N = U$ $G/N = U \times V$ where U $U \leq Z(G/N)$ is elementary abelian and V is an extra-special group of exponent p. This allows us to construct an automorphism of order p not fixing the commutator subgroup in finite p-groups with a cyclic center. As a consequence, we deduce that finite p-groups of coclass 4 and 5 admit a non-inner automorphism of order p ($p \geq 5$) We also prove the conjecture for an odd order nonabelian p-group G with cyclic center satisfying $C_G(G^p\gamma_3(G)) \cap Z_3(G) \leq Z(\Phi(G))$.

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Filial rings on quotient divisible Abelian groups

Ekaterina Kompantseva, Financial University, Moscow
and Thi Quynh Trang Nguyen, FPT University

A ring on an Abelian group G is a ring, whose additive group is isomorphic to G.

L. Fuchs fomulated the problem of describing Abelian groups that admits regular ring structure. Filial rings are a generalization of regular rings.

In the present paper, additive groups of regular rings and groups on which any associative ring is filial are described in the class of quotient divisible Abelian groups of rank 1.

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A generalization of a result on the sum of element orders of a finite group

Mihai-Silviu Lazorec, Alexandru Ioan Cuza University of Iasi, Romania

Let G be a group of order n and H be a subgroup of order m of G. Denote by $\psi_H(G)$ the sum of element orders relative to H of G. It is known that if G is nilpotent, then $\psi_H(G) \leq \psi_{H_m}(G)$, where H_m is the unique subgroup of order m of C_n. We show that this inequality does not hold for infinitely many finite solvable groups.

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Groups having all elements off a normal subgroup with prime power order

Mark Lewis, Kent State University

We consider a finite group G with a normal subgroup N so that all elements of $G \setminus N$ have prime power order. We prove that if there is a prime p so that all the elements in $G \setminus N$ have p-power order, then either G is a p-group or $G = PN$ where P is a Sylow p-subgroup and $(G, P, P \cap N)$ is a Frobenius-Wielandt triple. We also prove that if all the elements of $G \setminus N$ have prime power orders and the orders are divisible by two primes p and q, then G is a $\{p, q\}$-group and G/N is either a Frobenius group or a 2-Frobenius group. This builds on the classification by Higman of solvable groups where all elements have prime power order. If all the elements of $G \setminus N$ have prime power orders and the orders are divisible by at least three primes, then all elements of G have prime power order and G/N is nonsolvable. The nonsolvable groups where all elements have prime power order have been classified by Brandl extending the work of Suzuki who determined the simple groups with all elements having prime power order.

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Punctured groups for exotic fusion systems

Justin Lynd, University of Louisiana at Lafayette

The fusion system of a finite group G at a prime p is a category whose objects are the subgroups of a fixed Sylow p-subgroup S, and where the morphisms are the conjugation homomorphisms induced by the elements of G. The notion of a saturated fusion system is abstracted from this standard example and leads to exotic examples, namely fusion systems not realizable by a finite group in the above fashion. Given an exotic fusion system, one might want to ask: how close is it to being realizable by a finite group? This leads to the study of punctured groups, which are the most group-like objects that one can hope to associate with an exotic fusion system. It turns out that some exotic systems have them, while some do not. I plan to give background on these topics, look at some examples such as the Benson-Solomon exotic fusion systems at the prime 2, and mention an application of the existence of a punctured group to the topology of classifying spaces. This is joint work with Assaf Libman and Ellen Henke.

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Central products and the Chermak-Delgado lattice

Ryan McCulloch, Elmira College

The Chermak-Delgado lattice of a finite group G is a modular, self-dual sublattice of the lattice of subgroups of G. In this talk we consider the Chermak-Delgado lattice of a central product. This is joint research with William Cocke.

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Generalizing the Chermak-Delgado measure

Arturo Magidin, University of Louisiana at Lafayette

Let G be a finite group, H a subgroup. The Chermak-Delgado measure of H in G is $m(H) = |H|\,|C_G(H)|$. The subgroups with maximum measure form a sublattice of the lattice of subgroups of G with many nice properties.

In joint work with Elizabeth Wilcox, Luise-Charlotte Kappe, and William Cocke, we are looking at generalizations that involve marginal subgroups. I will discuss the idea, some examples, the obstacles that arise to a direct generalization, and some of the ways in which they can be overcome.

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Bounds on the size of Schur multiplier of p-groups with graph theory

Tony Nixon Mavely, IISER Thiruvananthapuram

The Schur multiplier of a given group has been of considerable interest to mathematicians, and thus determining its size is a problem that has generated significant interest. For special groups of rank k, the bound on the size of the Schur multiplier was given for two extreme values of k. We provide a bound for the size of the Schur multiplier for all special groups of rank between the given extremes using a bound on the number of triangles for a given graph. We also improve the upper bound for the size of the Schur multiplier of groups with nilpotency class greater than or equal to 3. As a corollary, we also improve the bounds for the size of the Schur multiplier for groups of given coclass r.

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The number of irreducible characters in a Brauer block

J. Miquel Martínez, Universitat de Valencia

Let G be a finite group, p a prime and B a Brauer p-block with defect group D. Exploring properties of D through the knowledge of some invariants of B is a classical problem in character theory. Let $k(B)$ denote the number of irreducible characters in B. We discuss certain results concerning the possible structures of D for small values of $k(B)$. This is joint work with Noelia Rizo and Lucia Sanus.

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On the fields of values of irreducible characters

Juan Martínez Madrid, Universitat de Valencia

There are many results that relate the structure of a group with the fields of values of its irreducible characters. We will begin by reviewing some classical results. Then, we will focus on studying the maximum number of times that an extension of the rational numbers appears as the field of values of an irreducible character.

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On the solubilizer of an element in finite groups

Carmine Monetta, University of Salerno

The solubility graph associated with a finite group G is a simple graph whose vertices are the elements of G, and there is an edge between two distinct vertices if and only if they generate a soluble subgroup.

The aim of this talk is to present some new results related to the solubility graph, underlining how graph properties affect the structure of the group G. In particular, we will focus on the set of neighbors of a vertex x, which we call the *solubilizer* of x in G, investigating both arithmetic and structural properties of this set.

This is a joint work with Banafsheh Akbari and Costantino Delizia.

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Zeta functions of the joint algebras over finite fields

Tung Nguyen, Western University

A natural model of a network of oscillators is a G-circulant matrix where G is a (cyclic) group. We observe that the set of all G-circulant matrices is naturally isomorphic to the group algebra k[G] of G. Consequently, the Artin-Wedderburn theorem for k[G] allows us to describe the eigenspectrum of these circulant networks concretely. Due to this explicit description, many problems involving circulant matrices have closed-form or analytical solutions. In our investigation of the non-linear dynamics on multilayer networks of oscillators, the joins of circulant graphs appear quite often, and they provide new exciting phenomena. This naturally leads us to investigate the joint algebra, an algebra obtained by joining several group algebras. We will introduce this joint algebra, some of its properties, and then describe its zeta function when the base field is a finite field. This is joint work with Sunil Chebolu, Jon Merzel, Jan Minac, Lyle Muller, Federico Pasini, and Nguyen Duy Tan.

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Iterated commutators of powers of an element

Jeffrey Riedl, University of Akron

Let x, y be elements in a group G. Let $d(k)$ denote the iterated commutator element $[\ldots[[x,y],y],\ldots,y]$ where y appears k times. This defines a sequence $\mathcal{D} = \{d(0), d(1), d(2), \ldots\}$. Let m be a positive integer. We present a formula and some techniques for expressing the element $[\ldots[[x,y^m],y^m],\ldots,y^m]$ as a product of elements of $\mathcal{D}$ in a convenient way. We apply these techniques to gain information about how certain normal subgroups of the regular wreath product $W = \mathbb{Z}_{p^2} \wr \mathbb{Z}_{p^2}$ are permuted in a natural action of the group of units $U(p^2)$ via automorphisms on W.

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On existence of minimal generating sets and maximal independent sets in groups and the additive semigroup of integers

Marshal Sampson, Akwa Ibom State University, Ikot, Akpaden, Nigeria.

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A survey about F

Mark Sapir, Vanderbilt University

I am going to talk about applications of the Thompson group F to links and knots discovered by Vaughan Jones. Joint work with Gili Golan-Polak.

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The 2-parts of relative character degrees

Chris Schroeder, Binghamton University

Let G be a finite group. The following result, proved by Higgs and Moretó, was used by Navarro and Tiep in their proof of Brauer's height zero conjecture for 2-blocks of maximal defect: suppose Z is a normal subgroup of G and θ is a G-invariant irreducible character of Z. If the relative degree $\chi(1)/\theta(1)$ is odd for every irreducible character χ of G lying over θ, then G/Z is solvable. The original proof of this result used the language of projective representations. We give a character theoretic proof and report progress on a conjecture extending this result to the case where the relative degrees are not divisible by 4. Our work extends to the projective setting a result of Lewis that if G is nonsolvable and $4 \nmid \chi(1)$ for all irreducible characters χ of G, then $G \cong A_7 \times S$ with S solvable.

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Frobenius groups leading to branch groups

Zoran Sunic, Hofstra University

A Frobenius group is a finite, transitive, permutation group in which no nontrivial group element fixes more than one point, but some nontrivial elements do fix a point (thus, the action is not free). Starting with various Frobenius groups we construct families of groups of tree automorphisms with remarkable properties. The previous construction (by Alejandra Garrido and the author) of the first family of regular branch groups with infinite rigid kernels can be understood as an instance of this approach, as can some examples constructed by Rachel Skipper. The best known example that fits in this general construction is the Hanoi Towers group. The understanding of these older examples through the lens of Frobenius groups yields new examples and results.

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Schur's Exponent Conjecture and related problems

Viji Thomas, Indian Institute of Science Education
and Research Thiruvananthapuram

We will discuss Schurs exponent conjecture, and discuss the progress made towards the conjecture in recent years. Finally we will describe our contribution towards this conjecture. Time permitting we will also discuss some problems in graph theory and apply it to obtain the size of the Schur Multiplier for some classes of groups.

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Equal coverings of finite groups

Andrew Velasquez-Berroteran, Adelphi University

A covering of a group is collection of proper subgroups whose set-theoretic union is the group. An equal covering is a covering where all proper subgroups are of the same order. We will look at some theorems and a look at GAP code relevant to finding which sort of finite groups have an equal coverings and which do not. Some general history of the research done in coverings within the past century will also be mentioned.

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