§5.5 Average Value

Find the average value of each function on the given interval.

a) $f(x) = 10x - x^2$ on the interval $[0, 2]$

b) $f(\theta) = 11\sec^2(\theta/4)$ on the interval $[0, \pi]$

c) $h(x) = 7\cos^4(x)\sin(x)$ on the interval $[0, \pi]$

Consider the function

$$f(x) = 3\sqrt{x}.$$

a) Find the average value f_{ave} of f on the interval $[0, 16]$.

b) Find all values c such that $f_{\text{avg}} = f(c)$.

c) Sketch the graph of f and, in the same picture, a rectangle whose area is the same as the area under the graph of f.

Consider the function

$$f(x) = (x-5)^2.$$

a) Find the average value f_{ave} of f on the interval $[4, 7]$.

b) Find all values c such that $f_{\text{avg}} = f(c)$.

c) Sketch the graph of f and, in the same picture, a rectangle whose area is the same as the area under the graph of f.

Consider the function

$$f(x) = 9\sin(4x).$$

a) Find the average value f_{ave} of f on the interval $[-\pi, \pi]$.

b) Find all values c such that $f_{\text{avg}} = f(c)$.

c) Sketch the graph of f and, in the same picture, a rectangle whose area is the same as the area under the graph of f.

Find all numbers b such that the average value of

$$f(x) = 7 + 10x - 9x^2$$

on the interval $[0, b]$ is equal to 8.

The velocity v of blood that flows in a blood vessel with radius R and length L at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta L}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) over the interval $0 \leq r \leq R$.