§4.3 Fundamental Theorem

Calculate the following integrals using Part II of the Fundamental Theorem of Calculus.

a) $\int_{-1}^{2} (x^3 - 4x)\, dx$

b) $\int_{4}^{9} \sqrt{x}\, dx$

c) $\int_{\frac{\pi}{6}}^{\pi} \sin(\theta)\, d\theta$

d) $\int_{0}^{1} (x+3)(x-6)\, dx$

e) $\int_{1}^{16} \frac{x-3}{\sqrt{x}}\, dx$

f) $\int_{-2}^{1} x^{-4}\, dx$

You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time t is given by $s(t)$. Which of the following represent your average velocity for that time interval:

i) $\dfrac{1}{b-a}\displaystyle\int_a^b v(t)\,dt$

ii) $\dfrac{s(b)-s(a)}{b-a}$

iii) $v(c)$ for at least one c between a and b.

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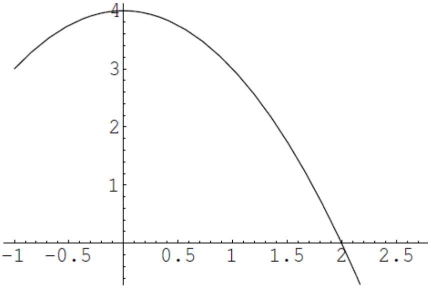
a) i, ii, and iii

b) i only

c) i and ii only

Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)\,dt$. Then for $0 < x < 2$, $g(x)$ is

a) increasing and concave up.
b) increasing and concave down.
c) decreasing and concave up.
d) decreasing and concave down.

True or **False**

If f is continuous on the interval $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x).$$

True or **False**

Let f be continuous on the interval $[a, b]$. There exist two constants m and M, such that

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a).$$

True or **False**

If $f'(x) = g'(x)$, then $f(x) = g(x)$.

Calculate the following derivatives using Part I of the Fundamental Theorem of Calculus.

a) $\dfrac{d}{dx}\displaystyle\int_0^x \frac{\mathrm{dt}}{1+t^2}$

b) $\dfrac{d}{dx}\displaystyle\int_0^{x^2} \frac{\mathrm{dt}}{1+t^2}$

c) $\dfrac{d}{dx}\displaystyle\int_{-x^2}^{x^2} \frac{\mathrm{dt}}{1+t^2}$

d) $\dfrac{d^2}{dx^2}\displaystyle\int_0^x \frac{\mathrm{dt}}{1+t^2}$

e) $\dfrac{d}{dx}\displaystyle\int_1^{\tan(x)} t^{10}\cos t\, dt$

f) $\dfrac{d}{dx}\displaystyle\int_{x^3}^{x^5+1} \frac{1}{t}\, dt$

If f is continuous and $f(x) < 0$ for all x in the interval $[a, b]$, then $\int_a^b f(x)\,dx$

a) must be negative.
b) might be zero.
c) not enough information.

If f is a differentiable function, then $\int_0^x f'(t)\,dt = f(x)$

a) Always.
b) Sometimes.
c) Never.

A sprinter practices by running various distances back and forth along a straight line. Her velocity at t seconds is given by the function $v(t)$. What does $\int_0^{60} |v(t)|\, dt$ represent?

a) The total distance the sprinter ran in one minute.
b) The sprinter's average velocity in one minute.
c) The sprinter's distance from the starting point after one minute.
d) None of the above.

Water is pouring out of a pipe at the rate of $f(t)$ gallons per minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$ minutes. The amount of water you collect can be represented by

a) $\int_2^4 f(x)\,dx$
b) $f(4) - f(2)$
c) $(4 - 2)f(4)$
d) The average of $f(4)$ and $f(2)$ times the amount of time the elapsed.

If f is continuous on the interval $[a, b]$ then

a) $\int_a^b f(x)\,dx$ is the area bounded by the graph of f, the x-axis, and the lines $x = a$ and $x = b$.

b) $\int_a^b f(x)\,dx$ is a number.

c) $\int_a^b f(x)\,dx$ is an antiderivative of $f(x)$.

d) $\int_a^b f(x)\,dx$ may not exist.

If $\int_a^b f(x)\,dx = b^3 - a^3$)for all numbers a and b,

then what is $\int_a^b f'(x)\,dx$?

If $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = x^3 - 1$, then what is $\int_a^b f'(x)\,dx$?

If $\int_a^b f(u(x))u'(x)\,dx = (2/3)(b^2+1)^{3/2} - (2/3)(a^2+1)^{3/2}$ for all numbers a and b, then what might $f(x)$ and $u(x)$ be? Are they unique?