

§3.2 Mean Value Theorem

True or False

There is a function $f(x)$ which is

- ▶ continuous on $[0, 1]$,
- ▶ has average rate of change of 3 on $[0, 1]$, and
- ▶ **never** has instantaneous rate of change of 3 on $(0, 1)$.
(That is, there's no c in $(0, 1)$ with $f'(c) = 3$.)

True or False

There is a function $f(x)$ which is

- a) continuous on $[0, 1]$,
- b) differentiable on $(0, 1)$,
- c) has average rate of change of 3 on $[0, 1]$, and
- d) **never** has instantaneous rate of change of 3 on $(0, 1)$.
(That is, there's no c in $(0, 1)$ with $f'(c) = 3$.)

If the given function satisfies the hypotheses of the Mean Value Theorem on the given interval, then find all numbers c which satisfy the conclusion of the Mean Value Theorem.

a) $f(x) = 3x^2 + 2x + 5$ on $[-1, 1]$.

b) $g(x) = x^3 + x - 1$ on $[0, 2]$.

c) $h(x) = \frac{x}{x+2}$ on $[1, 4]$.

d) $k(x) = (x - 2)^{-2}$ on $[1, 4]$.

On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt.

Which of the following describes the situation?

- a) The booth attendant does not have enough information to prove that the driver was speeding.
- b) The booth attendant can prove that the driver was speeding during their trip.
- c) The driver will get a ticket for a lower speed than their actual maximum speed.

True or False

An athlete is running back and forth along a straight path. She finishes her run at the place where she began. There must be at least one moment, other than the end of the race, where she was at a complete stop.

Two runners start a race at the same moment and finish in a tie. What must be true?

- a) At some point during the race the two runners were not tied.
- b) The runners' speeds at the end of the race must have been exactly the same.
- c) The runners must have had the same speed at exactly the same time at some point in the race.
- d) The runners had to have the same speed at some moment, but not necessarily at exactly the same time.

Show that for all values a and b

$$|\sin(a) - \sin(b)| \leq |a - b|$$

Suppose that $3 \leq f'(x) \leq 5$ for all values of x . Show that

$$18 \leq f(8) - f(2) \leq 30$$

Show that the polynomial

$$f(x) = 1 + 2x + x^3 + 4x^5$$

has exactly one real root.