F'14 Final (Solutions)

Problem 1(10 points) Alexandra, Belinda, Chester, Dippy, Eddy, and Fonzie are trying to divide a piece of land using claim and challenge. They choose in order from A to F. In the first round, everybody challenges. In the second round, Dippy gets the piece. In the fourth round, Belinda makes the claim and Eddy gets the piece.

A	B	C	D	E	F
3			2	4	1

(2 points)
(a) Who gets the piece in the first round?

F

(2 points)
(b) Based on Dippy getting the piece, who could have challenged in the second round, including the original claim?

A, B, C, D

(3 points)
(c) How many challenges are made in the third round, including the original claim?

only A

(3 points)
(d) Who is left in the fifth round?

B & C

Problem 2 (15 points)
50 seats are to be divided up between five states, A, B, C, D, and E. Use Hamilton's method to distribute the seats amongst the states.

State	A	B	C	D	E	Total
Population	83	116	74	107	120	500
No. of seats: 50		Standard divisor:	10			
Exact Quota	8.3	11.6	7.4	10.7	12	XXXXX
Lower Quota	8	11	7	10	12	48
Frac Part	.3	.6	.4	.7	0	XXXXX
Surplus		1		1		2
Total	8	12	7	11	12	50

$$\begin{array}{r} 83 \\ +116 \\ \hline 199 \\ +74 \\ \hline 273 \\ +107 \\ \hline 380 \\ +120 \\ \hline 500 \end{array}$$

Problem 3 (15 points)

Consider a deck of cards that has 10 ranks (1 through 10) and seven suits (hearts, diamonds, clubs, spades, circles, squares, and moons). Thus there are 70 cards in the deck. (You may leave your answers in terms of products and sums of factorials, ${}_nP_k$, ${}_nC_k$ etc. For example, answers of the form 6^5 or ${}_6C_3 \times {}_3C_1$ are acceptable.)

(3 points) **(a)** Suppose we draw a **5-card** hand from this deck. How many three of a kinds are possible? (A three of a kind is a hand where 3 cards are of the same rank and the other two cards are each a different ranks. For example, 4, 4, 4, 8, 9.)

$${}_{10}C_1 \times {}_7C_3 \times {}_9C_2 \times {}_7C_1 \times {}_7C_1$$

(3 points) **(b)** Suppose we draw a **5-card** hand from this deck. How many straights are possible?(A straight is a hand where the ranks of the cards are in order, for example, 3,4,5,6,7 of any suit. Assume straight flushes are included as straights.)

$${}_6C_1 \times ({}_7C_1)^5$$

(3 points) **(c)** Suppose we draw a **5-card** hand from this deck. What is the **probability** that the sum of the ranks in the hand is 50?

$$\frac{{}_7C_5}{{}_{70}C_5}$$

(3 points) **(d)** Suppose we draw a **6-card** hand from this deck. Define a 3-pair hand as getting 3 pairs, each of a different ranks. For example, 3, 3, 4, 4, 8, 8. How many 3-pair hands are possible?

$${}_{10}C_3 \times ({}_7C_2)^3$$

(3 points) **(e)** Suppose we draw a **10-card** hand from this deck. How many 10-card hands are there where the sum of the ranks is 10?

0

Problem 4 (15 points)
(5 points)
(a) Five people (labeled A through F, in order) are in this weighted voting system: [10; 8, 5, 3, 2, 1, 1]. Is this system in anarchy, is it non-functional, or neither?

anarchy

(3 points)
(b) List all possible values for q that satisfy the quota restriction for the weighted voting system [q ; 8, 4, 3, 1].

$w=16$, so $8 < q \leq 16$

(5 points)
(c) Consider the weighted voting system: [q ; 10, 6, 4, 4]. Is there a possible q value satisfying the quota restriction for which voter A has veto power?. If so, list only one. If not, say "no."

yes, $q=15$ (for example)

(2 points)
(d) Consider the weighted voting system [q ; 8, 4, 1, 1]. Find the quota restriction and state the q-value satisfying the quota restriction that makes voter A a dictator.

$7 < q \leq 14$ $\quad q=8$

Problem 5(10 points)
(3 points)
(a) Using the following preference schedule, who wins the election using the plurality method?

	Num ballots			
Choice	5	5	9	2
1st	A	A	B	C
2nd	D	B	C	A
3rd	C	C	A	D
4th	B	D	D	B

A

(2 points)
(b) If another voting method produced a different winner, would the majority criterion be violated? Explain.

no. Nobody has a majority, so the maj. crit. can't be violated.

(2 points)
(c) Who would be the first candidate eliminated using the plurality with elimination method?

D

(3 points)
(d) If you are using the pairwise comparison method and comparing A and B, who would get the point?

A

Problem 6 (15 points)
Sleepy, Bashful, Sneezy, and Doc have inherited some valuable pieces of fruit from a woman they used to know. Carry out the division of these objects between them using the sealed bids method.

~~1500~~

	Sleepy	Bashful	Sneezy	Doc
Apple	$1000	$1200	$1500	$700
Peach	$500	$700	$900	$800
Kiwi	$700	$500	$500	$500
Pear	$600	$800	$300	$400
Total Value	2800	3200	3200	2400
Fair Share	700	800	800	600
Allocated	700	800	2400	0
Difference	0	0	-1600	600
Surplus= 1000				
Surplus Share	250	250	250	250

Summary

Item(s)	kiwi	pear	apple + peach	—
Item's Value	700	800	2400	—
Cash	250	250	-1350	850
Net Total	950	1050	1050	850

Problem 7 (20 points)

Two 6-sided dice are rolled and a coin is flipped. One of the dice and the coin are fair and colored red, and the other die is colored blue and weighted with weights according to the following table:

face of die	1	2	3	4	5	6
probability	.2	.3	.1	.1	.1	.2

For all parts, consider the random experiment of rolling both dice and flipping the coin. Leave all answers as sums and products.

(6 points)

(a) Consider the event X = Both dice land on 2 and the coin is heads. What is the probability of X?

$$\frac{1}{6} \times .3 \times \frac{1}{2}$$

(6 points)

(b) Consider the event M = The red die is a 1, the blue die is a 4, and the coin is tails. What is the probability of S?

$$\frac{1}{6} \times .1 \times \frac{1}{2}$$

(6 points)

(c) Consider the event A = either event X happens or event M happens. What is the probability of A?

$$(a) + (b)$$

(2 points)

(d) Consider the event S = The sum of the numbers shown on the red and blue dice is 2. How many outcomes are in S?

2

(Careful to count the coin! $(1,1,H)$ and $(1,1,T)$)

Problem 8(15 points) Mothra is making a password for his Binghamton gmail account. He must make a password that is 9 characters. He may use any of the lowercase letters or any of the uppercase vowels (A, E, I, O, U).

(3 points)

(a) How many possible passwords can Mothra make?

31^9

(4 points)

(b) How many passwords will have no uppercase letters?

26^9

(4 points)

(c) How many passwords will have no repeated letters (Assume uppercase letters and lowercase letters are different, i.e. AabcdEefg has no repeated letters)?

$31 \times 30 \times 29 \times 28 \times 27 \times 26 \times 25 \times 24 \times 23$

(4 points)

(d) How many passwords will contain the sequence "xmAs" (For example, jxmAsiEIl)?

6×31^5

Problem 9(10 points)A poll was taken among Math 130 students to see how many hours they spent studying for their Math 130 final.
(2 points) **(a)** Complete the table.

Number of hours	0	1	2	3	4	5
frequency	10	22	30	23	20	35
cumulative freq.	10	32	62	85	105	140

(5 points) **(b)** Calculate the five number summary.

min = 0 Q1 = 2 med = 3 Q3 = 4.5 max = 5

(3 points) **(c)** Write down the mean. (You need not compute it.)

$$\frac{0\times 10 + 1\times 22 + 2\times 30 + 3\times 23 + 4\times 20 + 5\times 35}{140}$$

Problem 10 (10 points)
(8 points)
(a) Calculate the Standard Deviation of the data set 5, 6, 7, 7, 5. (You may leave your answer in terms of the Variance).

$$\sqrt{\frac{4}{5}}$$

(2 points)
(b) True or False: If the variance of a data set is bigger than 1, then the variance is always greater than the standard deviation.

Problem 11(15 points) The scores on the Math 130 final are approximately normally distributed with mean $\mu = 72$ points, and standard deviation $\sigma = 4$ points.

(3 points)

(a) Approximately what percentage of the scores are higher than an 80?

2.5%

(3 points)

(b) What is the 16th percentile?

68

(3 points)

(c) About what percent of the scores are between 72 and 80?

$\frac{95}{2} = 47.5\%$

(3 points)

(d) Estimate the 75^{th} percentile score. (You do not need to compute the decimal).

$72 + .675 \times 4$

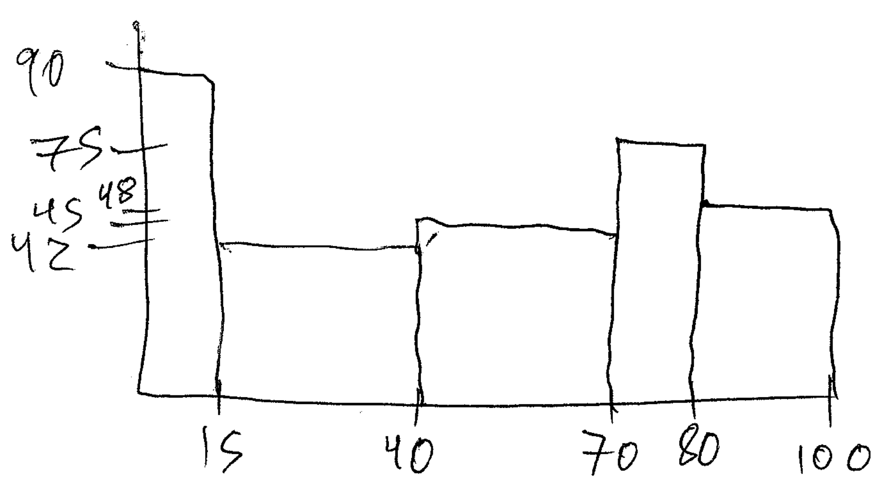
(3 points)

(e) About what percent of the scores are between 60 and 64?

$\frac{99.7 - 95}{2} = \frac{4.7}{2} = 2.35\%$

Problem 12 (15 points) The cookie monster eats a certain number of cookies each day and then records the results in this table. Create a bar graph for the number of cookies he eats.

cookies	[0, 15)	[15, 40)	[40, 70)	[70,80)	[80,100]
frequency	90	42	45	75	48



Problem 13(15 points) Some scientists are studying the fish population in the Susquehanna river. They captured, tagged and released 500 fish. A year later, they captured 200 fish of which 40 were tagged.

(3 points)

(a) Find $\hat{p}$, which is the estimate for the fraction of the fish population tagged.

$$\hat{p} = \frac{40}{200}$$

(3 points)

(b) Find the central estimate for the population.

$$\frac{40}{200} = \frac{500}{N} \Rightarrow N = \frac{500 \times 200}{40}$$

(3 points)

(c) Give a formula for the standard error (plug in the numbers that you have).

$$\text{st. error} = \sqrt{\frac{\frac{40}{200} \times \left(1 - \frac{40}{200}\right)}{200}}$$

(6 points)

(d) Assume the standard error is .03. Find the 99.7% confidence interval for the population of puppies. You may leave the inequality/interval as fractions.

$$\frac{40}{200} - 3 \times .03 \leq p \leq \frac{40}{200} + 3 \times .03$$

$$'' \quad \leq \frac{500}{N} \leq \quad ''$$

$$\frac{500}{\frac{40}{200} - 3 \times .03} \geq N \geq \frac{500}{\frac{40}{200} + 3 \times .03}$$

Problem 14 (2 points each)

(a) True or **False** The median of a set of numbers is always larger than the mean.

(b) True or **False** $100 = {}_{100}C_1$

(c) True or **False** Ann has a half chocolate, half vanilla cake worth $50 and she wants to do the cut and choose method. If Ann cuts the cake in half so that one half is all chocolate and the other is all vanilla, then she must like each flavor equally.

(d) True or **False** In any data set, you can estimate the 25th percentile value by $\mu - .675\sigma$.

(e) True or **False** The plurality method never violates the Independence of Irrelevant Alternatives criterion.

(f) True or **False** If a voter is removed from a winning coalition and the coalition becomes a losing coalition, we call that voter a dictator.

(g) True or **False** Putting on a scarf and putting on a pair of gloves is an example of independent events.

(h) True or **False** If you flip fair coin 99 times and get heads every time, then the probability of getting heads on your next flip is greater than .5.

(i) True or **False** The You Cut I Choose method is an example of a fair division that is always envy free.

(j) True or **False** In Webster's method, a quota of 4.5 would round up to 5.