Math 130 **Final Exam (Make Up)** **December 9, 2011**

Print your name and your section number and sign below, and read the instructions. Do not open the test until you are told to do so.

Name (printed): Solutions

Section:

Signature:

This test has 0 questions on 0 pages. The total number of points is 0.

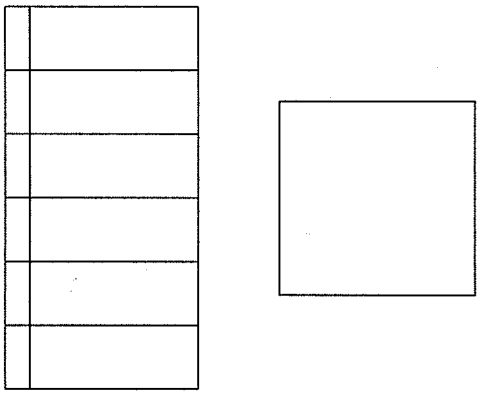
When the proctor says you may begin then check that you have a complete test.

Put all your answers in the spaces provided on these sheets. The backs of the test sheets are blank and may be used for scratch work. More scratch paper is available on request.

You must show all your work. You must show enough work to indicate how you got your answer. You will lose credit for incorrect statements or incorrect mathematical expressions. Neatness and clarity are important. You will lose credit if we cannot decipher your answer.

You will be graded on what you write in the space provided for your work. Cross out any scratch work, or label it as scratch. If your work is not in the space provided, indicate clearly where we may find it, and label it. Do not give two or more answers for the same problem.

Do not write inside this box.

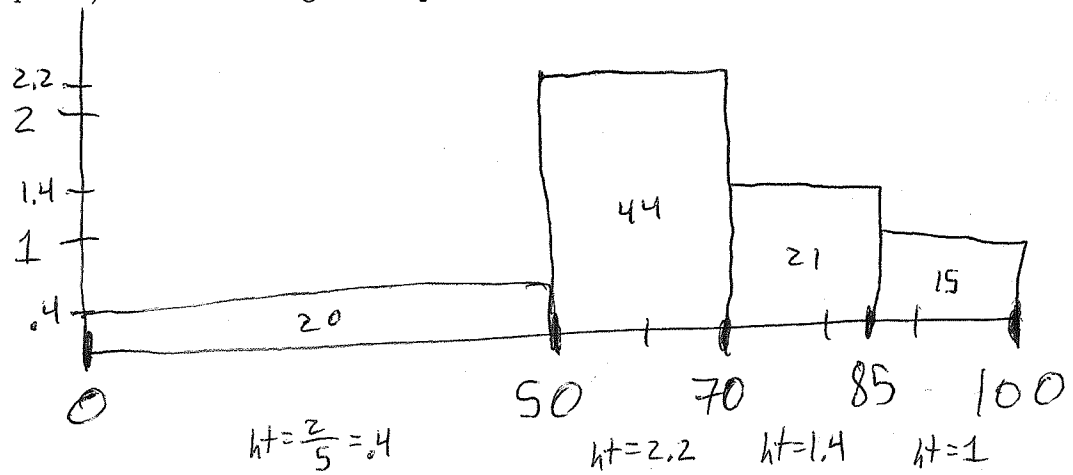


1. (25 points) **Histograms**

A study on the cost of textbook prices at the BU bookstore revealed the following data:

20% of books cost between \$0 and \$50.
44% of books cost between \$50 and \$70.
21% of books cost between \$70 and \$85.
15% of books cost between \$85 and \$100

(1) (10 points) Draw the histogram for prices of textbooks.



(2) (5 points) What percentage of textbooks sold for less than \$70?

64%

(3) (5 points) What percentage of textbooks sold for more than \$80?

7+15 = 22%

(4) (5 points) 53% of textbooks sold for less than what price?

$50 + \frac{3}{4} \times 20 = 50 + 15 = \$65$

24 × 7 = 168 (2) 26 × 7 = 182 (4) 27 × 4 = 108 (2)

2. (30 points) **Dispersion**

(10 points each) A dog breeder is measuring the weights (in pounds) of adult Pembroke Welsh Corgis. His data are presented in the following frequency table. The extra columns and rows are for your reference. It is not necessary to fill in every box in the table.

Weight (x)	20	23	24	26	27	29	Total	Average
Frequency (f)	1	3	7	7	4	2		
Cum. Freq.	1	4	11	18	22	24		
$x \cdot f$	20	69	168	182	108	58		

(1) Compute the five number summary of this data set.

Min = 20
Q1 = 24
Med = 26
Q3 = 26.5
Max = 29

Scratch work: median location: 12 & 13, so 26 & 26 weights

Lower half: locations 1–12, so Q1 location is 6&7: 24&24

Upper half: locations 13–24, so Q3 location is 18&19: 26&27

(2) Compute the mean of this data set.

$$\frac{\Sigma x \cdot f}{\Sigma f} = \frac{20+69+168+182+108+58}{24} = \text{no calculator}$$

(3) Compute the standard deviation of this data set.

way too hard w/o calculator

3. (30 points) **Statistical Inference I**
A zoologist wants to estimate the number of parrots in a Costa Rican rainforest. She captures 500 parrots and tags them. Later, she collects a sample of 250 parrots and observes that 49 of them are tagged.

(1) (15 points) Use the capture-recapture method to find the central estimate for the number of parrots in the rainforest.

$$\frac{49}{250} = \frac{500}{N} \Rightarrow N = \frac{250 \times 500}{49}$$

(2) (15 points) Construct a 68% confidence interval for the number of parrots in the rainforest.

$$\hat{p} = \frac{49}{250} \qquad \frac{\sigma}{n} \approx \sqrt{\frac{\hat{p} \times (1-\hat{p})}{n}} = \sqrt{\frac{\frac{49}{250} \times \frac{201}{250}}{250}}$$

$$\hat{p} - \frac{\sigma}{n} \leq \underbrace{p}_{\frac{500}{N}} \leq \hat{p} + \frac{\sigma}{n}$$

$$\rightsquigarrow \quad \frac{500}{\hat{p} - \frac{\sigma}{n}} \geq N \geq \frac{500}{\hat{p} + \frac{\sigma}{n}}$$

So
$$\frac{500}{\frac{49}{250} - \sqrt{\frac{\frac{49}{250} \times \frac{201}{250}}{250}}} \geq N \geq \frac{500}{\frac{49}{250} + \sqrt{\frac{\frac{49}{250} \times \frac{201}{250}}{250}}}$$

yeesh. They definitely had calculators in 2011.....

4. (25 points) **Statistical Inference II**
(5 points each) In a public opinion poll of 1200 likely voters prior to the 2012 presidential election, 675 people preferred candidate A and 525 people preferred candidate B.

(1) (10 points) Construct a 95% confidence interval for the percentage of people that prefer candidate A.

$\hat{p} = \frac{675}{1200}$ $\quad \frac{\sigma}{n} \approx \sqrt{\frac{675}{1200} \times \left(1 - \frac{675}{1200}\right)}{1200}$

So $\frac{675}{1200} - 2\sqrt{\frac{\frac{675}{1200} \times \left(1 - \frac{675}{1200}\right)}{1200}} \leq p \leq \frac{675}{1200} + 2\sqrt{\frac{\frac{675}{1200} \times \left(1 - \frac{675}{1200}\right)}{1200}}$

(2) (10 points) Construct a 95% confidence interval for the percentage of people that prefer candidate B.

$\frac{525}{1200} - 2\sqrt{\text{same as above}} \leq p \leq \frac{525}{1200} + 2\sqrt{\text{same}}$

(3) (5 points) Based on your answers in parts (1) and (2), is there a statistically significant difference between the support for candidate A and the support for candidate B? Explain.

$\hat{P}(A) - \hat{P}(B) = \frac{675 - 525}{1200} = \frac{150}{1200} = \frac{15}{120} = \frac{1}{8}$

Is $\frac{1}{8}$ bigger than $2\sqrt{\frac{\frac{675}{1200} \times \left(1 - \frac{675}{1200}\right)}{1200}}$?

If yes, then yes. If no, then no.

5. (25 points) **Two-Outcome Experiment**
A Major League Baseball player reaches base in 39% of his plate appearances. Give numeric answers for this problem. no

(1) (5 points) According to the binomial principle, what is the average number of times the player will reach base in 60 plate appearances?

$.39 \times 60$

(2) (5 points) According to the binomial principle, what is the standard deviation for the number of times the player reaches base in 60 plate appearances?

$\sqrt{60 \times .39 \times .61}$

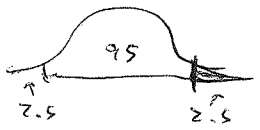
(3) (15 points) In the player's next 10 plate appearances, what is the probability that he will reach base exactly 4 times?

$${}_{10}C_4 \times (.39)^4 \times (.61)^6$$

6. (30 points) **Normal Distributions**
(10 points each) The heights of sunflowers are near normally distributed, with a mean of 9.50 feet and a standard deviation of 1.38 feet.

(1) What percentage of sunflowers are above 12.26 feet?

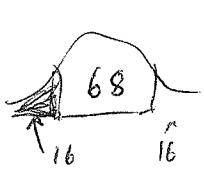
$$\frac{12.26 - 9.50}{2.76} = 2\sigma$$



2.5%

(2) What percentage of sunflowers are below 8.12 feet?

that's $\mu - \sigma$, so



16%

(3) 25% of sunflowers are above what height?

$Q3 \approx \mu + .675\sigma = 9.5 + .675 \times 1.38$

7. (35 points) **Counting**
In a lottery ticket a player marks four different numbers between (and including) 1 and 30. The order of the numbers on the ticket does not matter. For example, [1,5,4,13] and [5,13,4,1] are considered to be the same lottery ticket. In this problem, the answers in terms of products of numbers, permutations or combination are sufficient.

(1) (5 points) How many different lottery tickets are possible?

$_{30}C_4$

(2) (10 points) How many lottery tickets contain only even numbers?

$_{15}C_4$

(3) (10 points) How many lottery tickets contain exactly one number 3?

$_{29}C_3$

(4) (10 points) Four winning numbers are chosen. How many lottery tickets contain exactly three winning numbers?

$_4C_3 \times {}_{26}C_1$

[we didn't really do problems like this]

8. (30 points) **Sealed Bids**
Kim has just gotten divorced, and she is dividing her ex-husband's possessions among herself and her two sisters, Khloe and Kourtney. The possessions include a wedding ring, a Ferrari, and a pet monkey. She decides to use the Sealed Bids method. Their bids are given in the following table. Carry out the division.

	Kim	Khloe	Kourtney
Ring	12,000	10,000	18,000
Ferrari	132,000	160,000	143,000
Monkey	36,000	37,000	34,000
Total Bid	180,000	207,000	195,000
Fair Share	60,000	69,000	65,000
Allocated	0	197,000	18,000
Difference	60,000	-128,000	47,000
surplus = 21,000			
Surplus Share	7000	7000	7000

177
18
195

Summary

	Kim	Khloe	Kourtney
Item(s)	—	ferrari, monkey	ring
Value	0	197,000	18,000
Cash	67,000	-121,000	54,000
Net Total	67,000	76,000	72,000

9. (30 points) **Weighted Voting Systems**
Consider the weighted voting system $[q : 10, 7, 5, 3]$.

(1) (10 points) What are all the possible values for q that satisfy the quota restriction?

$$\frac{25}{2} < q \leq 25$$

(2) (20 points) Set $q = 18$. List all possible coalitions, circle the winning coalitions, underline all critical voters, and determine the power index for each voter.

(A) 10
(B) 7
(C) 5
(D) 3
(A, B) 17
(A, C) 15
(A, D) 13
(B, C) 12
(B, D) 10
(C, D) 8

(A, B, C) 22
(A, B, D) 20
(A, C, D) 18
(B, C, D) 15
(A, B, C, D) 25

$A: \frac{4}{10}$ $B: \frac{2}{10}$

$C: \frac{2}{10}$ $D: \frac{2}{10}$

10. (40 points) **True or False?** (4 points each)
Circle "True" at each statement that is always true, and circle "False" at each statement is not always true.

(a) True | False In a normal distribution, the third quartile lies exactly 0.675 standard deviations below the mean.

(b) True | False If the size of a data set is greater than 30, then the data set has a near-normal distribution.

(c) True | False If the size of a data set is even, then the median is an average of two numbers in the data set.

(d) True | False In a bar graph, the frequencies of data points are proportional to the areas of their respective bars in the graph.

(e) True | False The five number summary is a measure of dispersion in a data set.

(f) True | False ${}_nC_r \cdot r! = {}_nP_r$

(g) True | False The Minimum of the upper half of a data set is always the median of the data set.

(h) True | False In the Cut-and-Choose method, the cutter must divide his or her piece into equally-valued pieces.

(i) True | False The Borda Count method satisfies the majority criterion.

(j) True | False The only apportionment method among those that we studied that satisfies the quota criterion is the Huntington-Hill Method.