Math 130 – Test 3

Spring 2014

Wednesday, April 9, 2014

Name (printed): Solutions

Signature: ______________________

Section number: __________

Directions:
The test is one hour long. No phone, calculator, electronics, notes, talking to friends, etc. You may use only a pen or pencil. Absolutely no cheating!

No scrap paper! If you need some you may use the back side of this exam or ask someone who is proctoring the exam.

Read carefully. Show your work. Check your work.

Note: You may leave your answers in terms of products and sums of factorials, ${}_nP_k$, ${}_nC_k$ etc. For example, answers of the form 6^5 or ${}_6C_3 \times {}_3C_1$ are acceptable.

Do not turn the page until the professor and/or TA's say so.

Do not write below this line.

	Points		Points
1		5	
2		Total	
3			
4			

(The exam is out of 100 points.)

(25 points)

Problem 1

Someone decides to create a new deck of cards, called a super-deck. The super-deck has all the ranks in a normal deck, as well as one new rank, Princess, for a total of 14 ranks. The super-deck also has all of the suits in a normal deck, plus one new suit, the Moon, for a total of 5 suits (So we have 14x5=70 cards). For parts (a) and (b), you draw a 5-card hand (a hand is an unordered selection of cards).

(5 points)

(a) How many hands are possible?

$$_{70}C_5$$

(5 points)

(b) How many five-of-a-kind hands are possible? (A five-of-a-kind hand has 5 cards of the same rank).

$$\underbrace{_{14}C_1}_{\text{rank}} \times \underbrace{_5C_5}_{\text{suits}} \qquad (= 14)$$

(5 points)

(c) Suppose now that we draw a 7-card hand from the super-deck. How many flushes are possible? (A flush is a hand where all of the cards are of the same suit. In this question, straight-flushes are included as flushes.)

$$\underbrace{_5C_1}_{\text{suit}} \times \underbrace{_{14}C_7}_{\text{ranks}}$$

(10 points)

(d) Suppose that we draw another 7 card hand. How many Super-Full-Houses are there? (A Super-Full-House is a hand that consists of 5 cards of the same rank, along with two cards of a different rank.)

$$\underbrace{\underset{\text{(rank)}}{_{14}C_1} \times \underset{\text{(suits)}}{_5C_5}}_{\text{5-of-a-kind}} \times \underbrace{\underset{\text{(rank)}}{_{13}C_1} \times \underset{\text{(suits)}}{_5C_2}}_{\text{pair}}$$

$26+26+10=62$

(20 points)
Problem 2
Cleatus is trying to decide on a password for his computer. He is not very creative, so he decides to come up with a random 6-character password. The characters he can use are the lower case letters from a through z, the upper case letters from A through Z, and the digits 0-9 (note there are 26 letters in the alphabet).

(2 points)
(a) How many passwords can he make using lower case letters, upper case letters, and digits?

$$62^6$$

(3 points)
(b) How many passwords begin with three upper case letters?

$$26 \times 26 \times 26 \times 62 \times 62 \times 62$$

(5 points)
(c) How many passwords contain no vowels? (There are 5 lower case vowels, a,e,i,o,u, and 5 upper case ones, A,E,I,O,U).

$$52^6$$

(5 points)
(d) How many passwords contain at least one digit?

$$62^6 - \underbrace{\left(52^6\right)}_{\text{no digits}}$$

(5 points)
(e) How many passwords contain the word "green", all in lower case letters as written. (examples include 7green and greenH).

$$\underbrace{{}_2C_1}_{\text{green at front or back}} \times \underbrace{62}_{\text{other character}}$$

(25 points: 5 each)

Problem 3

(I) Consider the random experiment of rolling a red 4-sided die (numbered 1 through 4) and a green 8-sided die, (numbered 1 through 8). Both are fair.

(2 points)

(Ia) What is the size of the sample space?

$$4 \times 8 = 32$$

(3 points)

(Ib) Consider the event T: the number on the green die is five more than the number on the red die. What is the probability of T?

$(6,1), (7,2),$ or $(8,3)$, so $\frac{3}{32}$

(5 points)

(Ic) Consider the event S: the sum of the numbers on the red and green dice is 10. What is the probabilty of S?

$(2,8), (3,7), (4,6)$, so $\frac{3}{32}$

(II) Now consider the random experiment of rolling only the green 8-sided die from above, and a flipping weighted coin. This coin has a 3/4 probability of landing heads, and a 1/4 probability of landing tails. Answer the following questions:

(5 points)

(IIa) Consider the event R: The die shows an even number and the coin lands heads. What is the probability of R?

$$\left(\frac{1}{2}\right) \times \left(\frac{3}{4}\right)$$

(5 points)

(IIb) Consider the event Z: The die shows an odd number and the coin lands tails. What is the probabiltiy of Z?

$$\left(\frac{1}{2}\right) \times \left(\frac{1}{4}\right)$$

(5 points)

(IIc) What is the probability of Z and R occurring at the same time?

0 (trick question)

not burn ↓ $pr(S) = .92$ burn ↓ $pr(F) = .08$

[note: you'll get the same answer if you call "S" = "burn" & "F" = "not burn"]

(20 points)
Problem 4
Tony likes to make himself pizza for dinner. Sometimes he gets distracted while cooking, and so he burns his pizzas 8% of the time. One day he has a lot of company over and decides to make 10 pizzas.

(5 points)
(a) What is the probability that he does not burn any of the 10 pizzas?

$${}_{10}C_{10} \times (.92)^{10} \times (.08)^{0}$$

(5 points)
(b) What is the probability that he burns all 10 pizzas?

$${}_{10}C_{0} \times (.92)^{0} \times (.08)^{10}$$

(5 points)
(c) What is the probability that he burns ~~less~~ fewer than three of the pizzas?

0, 1, or 2 burned:
⇔ 10, 9, or 8 not burned: ${}_{10}C_{10} \times (.92)^{10} \times (.08)^{0} + {}_{10}C_{9} \times (.92)^{9} \times (.08)^{1} + {}_{10}C_{8} \times (.92)^{8} \times (.08)^{2}$

(5 points)
(d) What is the probability that he burns three or more of the pizzas?

1 − (answer to part (c))

(10 points: 2 each)
Problem 5

True or **False** ${}_nC_1 = {}_nP_1$ for any positive integer n.

True or **False** When flipping 6 fair coins in a row, you are just as likely to get T,T,T,T,T,T as ~~you~~ are H,T,H,T,H,T. (assume fair)

True or **False** ${}_nC_n = {}_nP_n$ for any positive integer n.

True or **False** If the probablity of an event E is P(E), the probability of E not occurring is $1/P(E)$.

True or **False** ${}_{40}P_{24}$ is a bigger number than ${}_{40}C_{24}$.