Math 130 **Test 3 (Make-up)** April 16, 2012

Print your name and your section number and sign below, and read the instructions. Do not open the test until you are told to do so.

Name (printed):

Section:

Solutions

Signature:

This test has 5 questions on 5 pages. The total number of points is 150.

When the proctor says you may begin then check that you have a complete test.

Put all your answers in the spaces provided on these sheets. The backs of the test sheets are blank and may be used for scratch work. More scratch paper is available on request.

You must show all your work. You must show enough work to indicate how you got your answer. You will lose credit for incorrect statements or incorrect mathematical expressions. Neatness and clarity are important. You will lose credit if we cannot decipher your answer.

You will be graded on what you write in the space provided for your work. Cross out any scratch work, or label it as scratch. If your work is not in the space provided, indicate clearly where we may find it, and label it. Do not give two or more answers for the same problem.

Do not write inside this box.

1	
2	
3	
4	
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1. (35 points) **Counting I**
Each driver's license issued by the state of Louisiana has an ID number consisting of 9 digits (0-9). For this problem, answers in terms of exponents, products, permutations, and combinations are acceptable.

(1) (5 points) How many Louisiana driver's license ID numbers are possible?

10^9

(2) (10 points) How many Louisiana driver's license ID numbers contain no repeated digits?

$10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

(3) (10 points) How many Louisiana driver's license ID numbers begin with three even numbers and end with two odd numbers? (Note: 0 is an even number!)

$5 \times 5 \times 5 \times 10 \times 10 \times 10 \times 10 \times 5 \times 5$

(4) (10 points) How many Louisiana driver's license ID numbers contain at most 7 3's?

First count opposite: "more than seven 3's".

disjoint → eight threes: ${}_9C_1 \times 9$ (choose which is not 3, & then 9 options there)
nine threes: only 1 way

So answer is: $10^9 - ({}_9C_1 \times 9 + 1)$

2. (40 points) **Counting II**
A gym class at Binghamton High School contains 8 girls and 8 boys. The teacher is creating a baseball team consisting of 9 students from the gym class. For this problem, answers in terms of exponents, products, permutations, and combinations are acceptable.

(1) (5 points) How many baseball teams are possible?

$_{16}C_9$

(2) (10 points) How many baseball teams contain at most 2 boys?

1 boy: $_8C_8 \times {}_8C_1$
2 boys: $_8C_7 \times {}_8C_2$
$\Rightarrow$ answer $= (_8C_8 \times {}_8C_1) + (_8C_7 \times {}_8C_2)$

(3) (5 points) The gym teacher wants to create a baseball lineup of 9 students from the gym class. How many lineups are possible? (Note: a lineup is a list of 9 students, indicating the order in which they will take their turns batting against the pitcher.)

$_{16}P_9$

(4) (10 points) How many baseball lineups start with a boy and then alternate between boys and girls?

$_8P_1 \times {}_8P_4 \times {}_7P_4$
first boy; 4 girls; 4 boys (either)

(5) (10 points) The best baseball player in the class is Jenny, and she prefers playing baseball with boys. How many lineups are there in which Jenny is the only girl?

$9!$ $\quad (= {}_9P_9)$

3. (30 points) **Probability I** (5 points each)
Chantal wants to conduct an experiment with three items: a 36-card deck of cards containing only the denominations 2 through 10 in each suit, a fair 12-sided die numbered 1 through 12, and a fair 6 sided die numbered 1 through 6. She decides to conduct the following experiment: **Draw one card from the 36-card deck, roll the 6-sided die, roll the 12-sided die.** Give numeric answers for this problem. NO

(1) What is the size of the sample space for this experiment?

$36 \times 6 \times 12$

(2) Consider the event E: Draw a heart, roll an even number on the 6-sided die, and roll an even number on the 12-sided die. How many outcomes are in the event E?

$9 \times 3 \times 6$

(3) What is the probability of the event E?

$$\frac{9 \times 3 \times 6}{36 \times 6 \times 12} \quad \left(= \frac{1}{16}\right)$$

(4) Consider the event F: The number rolled on the 12-sided die is the same as the number rolled on the 6-sided die. How many outcomes are in the event F?

$36 \times 6 \times 1$

(5) What is the probability of the event F?

$$\frac{36 \times 6 \times 1}{36 \times 6 \times 12} \quad \left(= \frac{1}{12}\right)$$

(6) Consider the event G: A king is drawn. How many outcomes are in the event G?

$4 \times 6 \times 12$

S = wear pr(S) = .6
F = not wear pr(F) = .4

4. (25 points) **Probability II**

Nicki has 10 wigs, 60% of which are pink. She is performing 9 concerts in the coming month. Before each concert, she will randomly select one wig out of her collection of 10 to wear during the performance. For this problem, answers in terms of exponents, products, permutations, and combinations are acceptable.

(1) (5 points) What is the probability that Nicki will wear a pink wig at all 9 shows?

$${}_9C_9 \times (.6)^9 \times (.4)^0$$

(2) (10 points) What is the probability that Nicki will wear a pink wig at exactly 6 shows?

$${}_9C_6 \times (.6)^6 \times (.4)^3$$

(3) (10 points) What is the probability that Nicki will wear a pink wig at more than 5 shows?

"more than 5" ⇔ "6, 7, 8, or 9"

So, it's: ${}_9C_6 \times (.6)^6 \times (.4)^3 + {}_9C_7 \times (.6)^7 \times (.4)^2 + {}_9C_8 \times (.6)^8 \times (.4)^1 + {}_9C_9 \times (.6)^9 \times (.4)^0$

5. (20 points) **True or False?** (4 points each)

Circle "True" at each statement that is always true, and circle "False" at each statement is not always true.

(a) (True) False If a six sided die is weighted so that the probability of rolling a 1 is 5 times the probability of rolling any other number, then the probability of rolling a 1 is 50%.

(b) True (False) ${}_nC_r$ is always greater than or equal to ${}_nP_r$, for any $0 \leq r \leq n$.

(c) True False In a standard 52-card deck of cards, there are more three-of-a-kind poker hands than two-pair poker hands. ${}_{13}C_1 \times {}_4C_3 \times {}_{12}C_2 \times ({}_4C_1)^2$ vs. ${}_{13}C_2 \times {}_4C_2 \times {}_4C_2 \times {}_{44}C_1$ need calculator...

(d) True (False) In a random experiment, all outcomes are equally likely.

(e) True (False) If E is an event in a random experiment, then the probability of the complement of E is $1/P(\text{E})$.