

Test 1
Math In Action (MAT 130)

Fall 2014

Wednesday, September 24th.

Name (printed): Solutions

Signature: _____

Section number: _____

Directions:

The test is one hour long. No phone, calculator, electronics, notes, talking to friends, etc. You may use only a pen or pencil. Absolutely no cheating!

No scrap paper! If you need some you may use the back side of this exam or ask someone who is proctoring the exam.

Read carefully. Show your work. Check your work.

Do not turn the page until the professor and/or TA's say so.

Do not write below this line.

	Points		Points
1		5	
2		6	
3			
4		Total	

The exam is out of 100 points.

Problem 1: The Athletes of the World Council is having a vote to decide on the best sport. The choices are Tennis, Basketball, and Soccer.

Choice	Num ballots			
	5	5	3	4
1st	S	T	T	B
2nd	B	B	S	S
3rd	T	S	B	T

(5 points each)

(a) Use the plurality method to rank the candidates.

T, S, B

(b) Use the borda count method to rank the candidates.

$$S: (5 \times 3) + (5 \times 1) + (7 \times 2) = 15 + 5 + 14 = 34$$

$$T: (9 \times 1) + (8 \times 3) = 9 + 24 = 33$$

$$B: (4 \times 3) + (10 \times 2) + (3 \times 1) = 12 + 20 + 3 = 35$$

so B, S, T

(c) Do your results from parts (a) and (b) violate the majority criterion? Explain.

no, no candidate even has a majority of 1st-place votes.

Problem 2: The fifty states of America are voting on their favorite holiday. Their choices are Independence Day, Christmas, and Thanksgiving.

Choice	Num ballots			
	12	15	10	13
1st	C	I	T	C
2nd	T	T	I	I
3rd	I	C	C	T

(5 points each)

(a) Use the pairwise comparison method to rank the candidates.

$$C \text{ vs } T: 25 - 25 : C + \frac{1}{2}, T + \frac{1}{2}$$

$$C \text{ vs } I: 25 - 25 : C + \frac{1}{2}, I + \frac{1}{2}$$

$$I \text{ vs } T: 28 - 22 : I + 1$$

$$\text{so } C:1, T:\frac{1}{2}, I:\frac{3}{2} : I, C, T$$

(b) Does the pairwise comparison method ever violate the condorcet criterion? Does your result in part (a) violate the condorcet criterion?

no no

(c) Will removing any of the non-winning candidates from the election in part (a) cause the independence of irrelevant alternatives criterion to be violated? Explain.

removing C is no problem.

removing T results in a tie between I & C, which technically "changes the outcome", so yes, IIA is violated. (But you could argue that I still "wins", so it's not violated..... We'll be careful to avoid ties....)

$$\begin{array}{r} 16 \\ 7 \\ \hline 248 \\ \hline 21 \end{array}$$

Problem 3: Consider the weighted voting system $[q: 9, 7, 7, 5, 3, 2, 1]$. Calculate the following. (5 points each)

(a) The total weight of the system.

$$34$$

(b) The q -values satisfying the quota restriction.

$$17 < q \leq 34$$

(c) A q -value that would be nonfunctional.

$$q = 35$$

(d) A q -value satisfying the quota restriction that would make a voter a dictator (if none exists, write "no").

no

Problem 4: Shake, Frylock, Meatwad, and Carl use a voting system to help keep their squad in order. Their weighted voting system is $[13; 7, 6, 2, 2]$. (Shake has weight 7, Frylock has weight 6, Meatwad has weight 2, and Carl has weight 2.)

(15 points)

(a) List all the coalitions and their weights; circle the winning coalitions, and underline every critical voter.

(S) 7	(MC) 4
(F) 6	<u>(SEM) 13</u>
(M) 2	<u>(SFC) 13</u>
(C) 2	(SMC) 11
<u>(SF) 13</u>	(FMC) 10
(SM) 9	<u>(SEMC) 17</u>
(SC) 9	
(FM) 8	
(FC) 8	

(5 points)

(b) Find the power index of each squad member. (Leave the numbers as fractions.)

$$S: \frac{4}{8} \quad F: \frac{4}{8} \quad M: \frac{0}{8} \quad C: \frac{0}{8}$$

Problem 5: The city council of Binghamton is voting to decide which superhero it would like to have protecting it. The choices are Wolverine, Batman, Superman, and Thor.

Choice	Num ballots			
	5	3	3	4
1st	S	W	T	T
2nd	B	B	B	W
3rd	T	S	S	B
4th	W	T	W	S

(15 points)

(a) Use the plurality with elimination method to rank the candidates. Show all work.

elim B:

	5	3	3	4
1st	S	W	T	T
2nd	T	S	S	W
3rd	W	T	W	S

elim W:

	5	3	3	4
1st	S	S	T	T
2nd	T	T	S	S
3rd	W	T	W	S

ranking is: S, T, W, B

(5 points)

(b) Assume Batman has won the election (this may or may not have happened in part (a)). Now, several voters change their preference by ranking Batman 4th instead of 1st. As a result, several people claim the monotonicity criterion has been violated. Are they correct? Explain.

no. monotonicity cares if people change their mind in favor of the winner, not against the winner.

Problem 6:

(2 points each)

True or **False** In the weighted voting system [12: 7, 5, 4, 1, 1], the voter with 7 votes has veto power.

True or **False** The pairwise comparison method does not violate any of the fairness criteria.

True or **False** In the weighted voting system [6: 7, 3, 1, 1], the voter with 7 votes is a dictator.

True or **False** The majority criterion is the only criterion that is satisfied by all four voting methods.

True or **False** A q-value of 7 would be an example of a q value that would place the system [q: 5, 4, 3, 3] in anarchy.

