Today’s plan:

- Section 1.2.7 : Rankings
- Section 1.3 : One person - Multiple votes; Two alternatives
Section 1.2.7 : Rankings
Elections are sometimes held for multiple offices at once, with each candidate interested in any position.
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For example, the Math Club could be electing president, vice-president and treasurer all at once.
Now it’s important to get not only the winner, but a ranking of the candidates.
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If we do the Borda count method or pairwise comparisons method, a ranking falls right out: most points to fewest points.
Example

- Math Club electing president, vice-president and treasurer.
Example

- Math Club electing president, vice-president and treasurer.
- Four candidates A, B, C, and D.
Example

They’ll be ranked and

- first place is president
Example

They’ll be ranked and
- first place is president
- second place is vice-president
Example

They’ll be ranked and

- first place is president
- second place is vice-president
- third place is treasurer
a) Get the ranking of candidates using the Borda count method.
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b) Get the ranking of candidates using the pairwise comparisons method.
Borda method:

- A gets 45 points
- B gets 57 points
- C gets 58 points
- D gets 40 points
Borda method:

We already got:

- A gets 45 points
- B gets 57 points
- C gets 58 points
- D gets 40 points
So, ranking with the Borda count method we get:

President: C
Vice-president: B
Treasurer: A
Pairwise comparison method:

- A gets 0 points
- B gets 3 points
- C gets 2 points
- D gets 1 points
Pairwise comparison method:

We already got:

- **A** gets 0 points
- **B** gets 3 points
- **C** gets 2 points
- **D** gets 1 points
So, ranking with the Pairwise comparison method we get:

- President: B
- Vice-president: C
- Treasurer: D
Remark

*The two methods produce completely different results.*
Plurality with elimination can yield a ranking, by placing:

▶ the candidate eliminated in the first round in last place
▶ the candidate eliminated in the second round in second to last place
▶ and so on...
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- and so on...
Remarks:

- In order to get a complete ranking we can’t just stop when someone has a majority.
Remarks:

- In order to get a complete ranking we can’t just stop when someone has a majority.
- We **have to** keep going until there are only two candidates.
Example

Get the ranking of the candidates in the Math Club election, using Plurality with Elimination.
Solution

We already got:

- **candidate D** is eliminated in the first round
- **candidate B** is eliminated in the second round
- **candidate A** is eliminated in the third round
- **candidate C** is the winner
Therefore, according to the plurality with elimination method we get:

- President: C
- Vice-president: A
- Treasurer: B
(Note: We didn’t talk about rankings using the Plurality Method, but it’s clear how to do it.)
Section 1.3 : One Person – Multiple Votes; Two Alternatives
Example (Motivating)

At a shareholders assembly, the owners of a company vote on some propositions:
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- Y/N: Approve the CFO’s annual report;
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- Y/N: Reappoint the accounting firm;
Example (Motivating)

At a shareholders assembly, the owners of a company vote on some propositions:

- Y/N: Approve the CFO’s annual report;
- Y/N: Reappoint the accounting firm;
- Y/N: Increase the CEO’s salary.
Each stockholder participates in these decisions with a **number of votes proportional to the number of stocks owned**.
Definition

A weighted voting system is a voting system in which

- Each voter holds a certain number of votes, his/her weight
Definition

A weighted voting system is a voting system in which

- Each voter holds a certain number of votes, his/her weight
- all votes are on two-alternative (Y/N) propositions
Example

The Kleen Car Wash Co. is owned by 4 shareholders, A, B, C, and D.
Example

- The Kleen Car Wash Co. is owned by 4 shareholders, A, B, C, and D.
- They own 40%, 30%, 20%, and 10%, respectively, of the company stock.
This is a weighted voting system where

- **A** has 4 votes
- **B** has 3 votes
- **C** has 2 votes
- **D** has 1 vote.
Question

Is it clear how many votes it should take to pass a proposition?
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*Is it clear how many votes it should take to pass a proposition?*

Definition

The number of votes needed to pass/reject a proposition is called the **quota** of the voting system.
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Is it clear how many votes it should take to pass a proposition?

Definition

The number of votes needed to pass/reject a proposition is called the quota of the voting system.

The quota depends on the situation.
Since the total number of votes is 10, the quota should be at least 6.
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If it’s less, say 5, then a proposition could be both passed and rejected.
Since the total number of votes is 10, the quota should be at least 6.

If it’s less, say 5, then a proposition could be both passed and rejected. A voting system which allows this contradiction is called **anarchy**.
It also makes no sense to have a quota bigger than 10:
It also makes no sense to have a quota bigger than 10: nothing could ever pass!
It also makes no sense to have a quota bigger than 10: nothing could ever pass! Such a system is called a **non-functional** voting system.
Since we are interested in functional systems which are not anarchic, we impose the following quota restriction:
Quota Restriction

In a weighted voting system with $n$ voters having weights $w_1, w_2, \ldots, w_n$, the quota $q$ is restricted by the inequalities

$$\frac{w}{2} < q \leq w$$

where $w = w_1 + w_2 + \cdots + w_n$ is the total weight of the system.
In our previous example:

\[ w = 4 + 3 + 2 + 1 = 10 \]

so, the quota restriction is \( 5 < q \leq 10 \).
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- The total weight is
  \[ w = 4 + 3 + 2 + 1 = 10. \]
- so, the quota restriction is
  \[ 5 < q \leq 10. \]
Careful! The bottom inequality is $<$ and the top is $\leq$. 
Definition

When the quota is

- just above half of the total weight, we have a **simple majority** voting system
Definition

When the quota is

- just above half of the total weight, we have a **simple majority** voting system
- equal to the total weight, we have a **unanimity** voting system;
Definition

When the quota is

- just above half of the total weight, we have a **simple majority** voting system
- equal to the total weight, we have a **unanimity** voting system; here it takes all voters to say “yes” for a proposition to pass
<table>
<thead>
<tr>
<th>Definition</th>
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<tr>
<td>2/3 of the total weight, we have a <strong>2/3-majority</strong> voting system</td>
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</table>
2/3 of the total weight, we have a 2/3-majority voting system. Simple majority and 2/3-majority voting systems are common.
To simplify notation, we write a voting system in the following format:

\[ [q : w_1, w_2, \ldots, w_n] \]

where

- \( n \) is the number of voters.
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- \( q \) is the quota.
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\[ [q : w_1, w_2, \ldots, w_n] \]

where

- \( n \) is the number of voters
- \( w_1, w_2, \ldots, w_n \) are the weights (\# of votes) of the voters
- \( q \) is the quota.

Usually the weights are listed in decreasing order.
Example

The Kleen Car Wash company has two types of propositions:
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- **special propositions** which deal with important things. These require a **2/3 majority** to pass.
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- **special propositions** which deal with important things. These require a **2/3 majority** to pass.
- **ordinary propositions**, which are the rest. These require only a **simple majority**.
Example

Describe the two voting systems, in the form

\[ [q : w_1, w_2, \ldots, w_n] \]
Solution

Ordinary propositions A simple majority quota is $q = 6$, so this system is

$$[6 : 4, 3, 2, 1]$$
Solution

Special propositions

- 2/3 of 10 is 6.67 but the quota must be an integer so we take $q = 7$. 
Solution

Special propositions

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- The special proposition voting system is therefore
  
  $[7 : 4, 3, 2, 1]$
Solution

Special propositions

- 2/3 of 10 is 6.67 but the quota must be an integer so we take $q = 7$.
- The special proposition voting system is therefore $[7 : 4, 3, 2, 1]$.

A has enough votes to stop a special proposition on her own!
Definition
We say that a voter has **veto power** if he/she has enough weight to **block** a proposition on his/her own.
Note, however, that A does not have enough weight to pass a proposition all by herself.
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**Definition**

We say that a voter is a **dictator** if they have enough weight to pass a proposition all by themselves.
Remarks:

- A dictator must have weight $w_i \geq q > w/2$ and therefore there can be no more than one dictator.
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- A dictator must have weight $w_i \geq q > w/2$ and therefore there can be no more than one dictator.
- Voting systems with a dictator are not so interesting.
- The dictator can do whatever they want.
Example

Consider the voting system \([8 : 10, 3, 2]\).
Example

Consider the voting system $[8 : 10, 3, 2]$. It has a dictator.
Example

Consider the voting system \([8 : 10, 3, 2]\).

- It has a dictator, namely the first voter
Example

Consider the voting system \([8 : 10, 3, 2]\).

- It has a dictator, namely the first voter
- The other two voters have no power in the system.
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Consider the voting system \([8 : 10, 3, 2]\).

- It has a dictator, namely the first voter
- The other two voters have no power in the system.

For all practical purposes this system is the same as

\([1 : 1, 0, 0]\).
Next time:

**Section 1.3.1**: Coalitions and Section 1.3.2: Critical voters; Power Index