Today’s plan:

- Section 1.2.2: Preference Ballots
- Section 1.2.3: Borda Count Method
Section 1.2.2: Preference Ballots and Introduction to Fairness Criteria.
Remarks:

- It might not be ideal to only record first choice votes.
Remarks:

- It might not be ideal to only record first choice votes.
- Maybe we want to know first choice, second choice, third choice, etc.
Remarks:

- It might not be ideal to only record first choice votes.
- Maybe we want to know first choice, second choice, third choice, etc.
- For that we introduce...
Preference Ballots
Preference Ballots

Definition

In a **preference ballot** the voters rank **all** or **some** of the candidates according to their preferences.
Example

In the Math Club election for president, the club members are asked to rank the four candidates A, B, C, and D, according to their preferences. The outcome is:
|---|---|---|---|---|
If we only look at first choice preference, we have:
8 - A, 5 - B, 7 - C, 0 - D.
If we only look at first choice preference, we have:
8 - A, 5 - B, 7 - C, 0 - D.

However, 4 of the 5 voters that rank B as their first choice, rank C as their second choice. (The other one ranks D as second choice.)
We organize data into the preference schedule.
We organize data into the **preference schedule**.

**Definition**

In the **preference schedule** each distinct ballot is listed only once, with the number of occurrences indicated on top.
<table>
<thead>
<tr>
<th>Choice</th>
<th>Number of ballots</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8     6     1     1     4</td>
</tr>
<tr>
<td>1st</td>
<td>A  C  C  B  B</td>
</tr>
<tr>
<td>2nd</td>
<td>B  D  D  D  C</td>
</tr>
<tr>
<td>3rd</td>
<td>C  B  A  C  D</td>
</tr>
<tr>
<td>4th</td>
<td>D  A  B  A  A</td>
</tr>
</tbody>
</table>
There’s no simple way to pick a winner, taking everything into account, and being fair.
There’s no simple way to pick a winner, taking everything into account, and being fair.
Different methods have been designed.
There’s no simple way to pick a winner, taking everything into account, and being fair.

Different methods have been designed.

We will study some of the most important ones.
Question

*What does fair mean?*
What does fair mean?

- There are four basic fairness criteria that a method may or may not satisfy.
Question

What does fair mean?

- There are four basic fairness criteria that a method may or may not satisfy.
- The majority criterion was the first.
It turns out that each voting method fails at least one of the four criteria.
It turns out that each voting method fails at least one of the four criteria.

Question

*Is there a fair voting method?*
Answer

NO!
Answer

NO! In 1952 Kenneth Arrow proved the General Impossibility Theorem, which says it’s impossible to have a voting method satisfying all four fairness criteria.
Answer

NO! In 1952 Kenneth Arrow proved the General Impossibility Theorem, which says it’s impossible to have a voting method satisfying all four fairness criteria. This helped earn him the 1972 Nobel prize in Economics.
Here he is:

Kenneth Arrow (1921 - )
Section 1.2.3 : Borda Count Method
In the **Borda count method** each candidate gets a certain number of points, depending on the ranking.
In the **Borda count method** each candidate gets a certain number of points, depending on the ranking.

The points for all the ballots are added up.
In the **Borda count method** each candidate gets a certain number of points, depending on the ranking.

The points for all the ballots are added up.

The candidate with the largest number of points is then the winner.
The basic version of the Borda count method:
The **basic version** of the Borda count method:

- The first-choice candidate gets as many points as there are candidates.
The **basic version** of the Borda count method:

- The first-choice candidate gets as many points as there are candidates.
- Second-choice gets one fewer point
The **basic version** of the Borda count method:

- The first-choice candidate gets as many points as there are candidates.
- Second-choice gets one fewer point
- and so on.
Find the winner of the Math Club
president election using the Borda
count method.

<table>
<thead>
<tr>
<th>Choice</th>
<th>Number of ballots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>A 8, C 6, C 1, B 1, B 4</td>
</tr>
<tr>
<td>2nd</td>
<td>B 8, D 6, D 1, D 1, C 4</td>
</tr>
<tr>
<td>3rd</td>
<td>C 8, B 6, A 1, C 1, D 4</td>
</tr>
<tr>
<td>4th</td>
<td>D 8, A 6, B 1, A 1, A 4</td>
</tr>
</tbody>
</table>
We have:

- first choice gets 4 points
- second choice gets 3 points
- third choice gets 2 points
- fourth choice gets 1 point.

Let’s do this computation. [On the board].
Remarks:

- Note that when we used the plurality method A was the winner.
Remarks:

- Note that when we used the plurality method A was the winner.
- Different methods may produce different results.
One of the drawbacks of the Borda count method is that it violates the majority criterion.
One of the drawbacks of the Borda count method is that it violates the majority criterion.

Example

- The Clearview City Council has 15 members.
One of the drawbacks of the Borda count method is that it violates the majority criterion.

Example

- The Clearview City Council has 15 members.
- They’re electing a president by Simple Borda Count method.
One of the drawbacks of the Borda count method is that it violates the majority criterion.

Example

- The Clearview City Council has 15 members.
- They’re electing a president by Simple Borda Count method.
- There are 3 candidates.
The preference schedule is:

<table>
<thead>
<tr>
<th>Choice</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Number of ballots</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>
The preference schedule is:

<table>
<thead>
<tr>
<th>Choice</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>A</td>
<td>C</td>
</tr>
</tbody>
</table>

Find the winner of the election.
Solution

Since there are three candidates,
- first place candidates get 3 points,
Solution

Since there are three candidates,

- first place candidates get 3 points,
- second place candidates get 2 points,
Solution

Since there are three candidates,

» first place candidates get 3 points,
» second place candidates get 2 points,
» third place candidates get 1 point.
Solution

Since there are three candidates,

- first place candidates get 3 points,
- second place candidates get 2 points,
- third place candidates get 1 point.

Let’s do the computation and find the winner. [On the board.]
Here:

- candidate A has a majority of first place votes, namely 8 out of 15.
Here:

- candidate A has a majority of first place votes, namely 8 out of 15.
- But the winner is B.
Here:

- candidate A has a majority of first place votes, namely 8 out of 15.
- But the winner is B.

This is a violation of the majority criterion: the criterion says A ought to win, but B won instead.
Basic Borda Count Method

In an election with $k$ candidates:

- the first place candidate in each ballot receives $k$ points
- The second place candidate receives $(k - 1)$ points
- and so on...
The candidate with the largest number of points is the winner.
Jean Charles de Borda (1733 - 1799)
There are variations of the Borda count method.

Example (Approval Voting)

Each voter just says “yes” or “no” for each candidate.
There are variations of the Borda count method.

Example (Approval Voting)

Each voter just says “yes” or “no” for each candidate.
- Each “yes” is worth 1 point.
There are variations of the Borda count method.

Example (Approval Voting)

Each voter just says “yes” or “no” for each candidate.
- Each “yes” is worth 1 point.
- Each “no” is worth 0 points.
The candidate with the largest number of points is the winner.