Today's plan:

- Section 1.2.2 : Preference Ballots
- Section 1.2.3 : Borda Count Method

Section 1.2.2 : Preference Ballots and Introduction to Fairness Criteria.

Remarks:

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- For that we introduce. . .

Preference Ballots

Preference Ballots

Definition

In a preference ballot the voters rank all or some of the candidates according to their preferences.

Example

In the Math Club election for president, the club members are asked to rank the four candidates **A**, **B**, **C**, and **D**, according to their preferences. The outcome is:

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. C
2nd. D
3rd. A
4th. B

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. B
2nd. D
3rd. C
4th. A

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. B
2nd. C
3rd. D
4th. A

Ballot
1st. B
2nd. C
3rd. D
4th. A

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. B
2nd. C
3rd. D
4th. A

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. A
2nd. B
3rd. C
4th. D

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. B
2nd. C
3rd. D
4th. A

Ballot
1st. C
2nd. D
3rd. B
4th. A

Ballot
1st. A
2nd. B
3rd. C
4th. D

- If we only look at first choice preference, we have:
  8 - **A**, 5 - **B**, 7 - **C**, 0 - **D**.

- If we only look at first choice preference, we have:
  8 - **A**, 5 - **B**, 7 - **C**, 0 - **D**.
- However, 4 of the 5 voters that rank **B** as their first choice, rank **C** as their second choice. (The other one ranks **D** as second choice.)

We organize data into the **preference schedule**.

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Definition

In the preference schedule each distinct ballot is listed only once, with the number of occurrences indicated on top.

Choice	**Number of ballots**				
	8	6	1	1	4
1st	A	C	C	B	B
2nd	B	D	D	D	C
3rd	C	B	A	C	D
4th	D	A	B	A	A

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st. A	1st. C	1st. C	1st. A	1st. A	1st. B	1st. C
2nd. B	2nd. D	2nd. D	2nd. B	2nd. B	2nd. D	2nd. D
3rd. C	3rd. B	3rd. A	3rd. C	3rd. C	3rd. C	3rd. B
4th. D	4th. A	4th. B	4th. D	4th. D	4th. A	4th. A

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st. B	1st. B	1st. A	1st. C	1st. A	1st. B	1st. A
2nd. C	2nd. C	2nd. B	2nd. D	2nd. B	2nd. C	2nd. B
3rd. D	3rd. D	3rd. C	3rd. B	3rd. C	3rd. D	3rd. C
4th. A	4th. A	4th. D	4th. A	4th. D	4th. A	4th. D

Ballot	Ballot	Ballot	Ballot	Ballot	Ballot
1st. A	1st. C	1st. C	1st. B	1st. C	1st. A
2nd. B	2nd. D	2nd. D	2nd. C	2nd. D	2nd. B
3rd. C	3rd. B	3rd. B	3rd. D	3rd. B	3rd. C
4th. D	4th. A	4th. A	4th. A	4th. A	4th. D

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- Different methods have been designed.

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- Different methods have been designed.
- We will study some of the most important ones.

Question

What does **fair** *mean?*

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- There are four basic **fairness criteria** that a method may or may not satisfy.
- The majority criterion was the first.

It turns out that each voting method fails at least one of the four criteria.

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Question

Is there a fair voting method?

Answer

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NO! *In 1952 Kenneth Arrow proved the General Impossibility Theorem, which says it's impossible to have a voting method satisfying all four fairness criteria.*

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NO! *In 1952 Kenneth Arrow proved the General Impossibility Theorem, which says it's impossible to have a voting method satisfying all four fairness criteria. This helped earn him the 1972 Nobel prize in Economics.*

Here he is:

Kenneth Arrow (1921 -)

Section 1.2.3 : Borda Count Method

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- The points for all the ballots are added up
- The candidate with the largest number of points is then the winner.

The **basic version** of the Borda count method:

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- The first-choice candidate gets as many points as there are candidates.

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- Second-choice gets one fewer point

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- Second-choice gets one fewer point
- and so on.

Example

Find the winner of the Math Club president election using the Borda count method.

Choice	**Number of ballots**				
	8	6	1	1	4
1st	A	C	C	B	B
2nd	B	D	D	D	C
3rd	C	B	A	C	D
4th	D	A	B	A	A

We have:

- first choice gets 4 points
- second choice gets 3 points
- third choice gets 2 points
- fourth choice gets 1 point.

Let's do this computation. [On the board].

Remarks:

- Note that when we used the plurality method **A** was the winner.

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- Different methods may produce different results.

One of the drawbacks of the Borda count method is that it violates the majority criterion.

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Example

- The Clearview City Council has 15 members.

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- They're electing a president by Simple Borda Count method.

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Example

- The Clearview City Council has 15 members.
- They're electing a president by Simple Borda Count method.
- There are 3 candidates.

The preference schedule is:

Choice	Number of ballots		
	8	4	3
1st	A	B	B
2nd	B	C	A
3rd	C	A	C

The preference schedule is:

Choice	Number of ballots		
	8	4	3
1st	A	B	B
2nd	B	C	A
3rd	C	A	C

Find the winner of the election.

Solution

Since there are three candidates,

- *first place candidates get 3 points,*

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Let's do the computation and find the winner. [On the board.]

Here:

- candidate **A** has a majority of first place votes, namely 8 out of 15.

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- candidate **A** has a majority of first place votes, namely 8 out of 15.
- But the winner is B.

This is a violation of the majority criterion: the criterion says **A** ought to win, but **B** won instead.

Basic Borda Count Method

In an election with k candidates:

- the first place candidate in each ballot receives k points
- The second place candidate receives $(k-1)$ points
- and so on...

The candidate with the largest number of points is the winner.

Jean Charles de Borda (1733 - 1799)

There are variations of the Borda count method.

Example (Approval Voting)

Each voter just says “yes” or “no” for each candidate.

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Example (Approval Voting)

Each voter just says “yes” or “no” for each candidate.

- Each “yes” is worth 1 point.
- Each “no” is worth 0 points.

The candidate with the largest number of points is the winner.