Section 3.1.5 continued: Counting poker hands

Example

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a) How many different poker hands are there?

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b) How many different poker hands are there, with all the cards from the $\clubsuit$ suit?

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A poker hand consists of 5 cards drawn from a 52-card deck.

a) How many different poker hands are there?

b) How many different poker hands are there, with all the cards from the ♣ suit?

c) How many different poker hands are there, with not all cards from the ♣ suit?

Solution

Order of the cards in a hand does not matter, so we're dealing with combinations.

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a) We want to choose 5 items out of 52. There are

$$_{52}C_5 = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$
$$= 2,598,960$$

different poker hands.

Solution

b) For the all-$\clubsuit$ hand, we choose 5 items out of 13.

Solution

b) For the all-♣ hand, we choose 5 items out of 13. There are

$$
{}_{13}C_5 = \frac{13 \times 12 \times 11 \times 10 \times 9}{5 \times 4 \times 3 \times 2 \times 1} = 1,287
$$

different poker hands with all cards ♣.

Solution

c) Finally, by the complement principle, there are

$$2,598,960 - 1,287 = 2,597,673$$

different poker hands where not all cards are $\clubsuit$.

Harder!

Example

How many "full house" poker hands are there? (Pair + triple)

How to count?

How to count? Imagine having to describe your hand. What info must you relate?

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1. Value of pair (e.g., 3, 7, J, A)

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1. Value of pair (e.g., 3, 7, J, A)
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3. Suits of the 2 pair cards

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1. Value of pair (e.g., 3, 7, J, A)
2. (Different) value of triple
3. Suits of the 2 pair cards
4. Suits of the 3 triple cards

How to count? Imagine having to describe your hand. What info must you relate?

1. Value of pair (e.g., 3, 7, J, A)
2. (Different) value of triple
3. Suits of the 2 pair cards
4. Suits of the 3 triple cards

Independent!

1. Value of pair
2. (Different) value of triple
3. Suits of the 2 pair cards
4. Suits of the 3 triple cards

1. Value of pair ${}_{13}C_1$ outcomes
2. (Different) value of triple
3. Suits of the 2 pair cards
4. Suits of the 3 triple cards

1. Value of pair ${}_{13}C_1$ outcomes
2. (Different) value of triple ${}_{12}C_1$
3. Suits of the 2 pair cards
4. Suits of the 3 triple cards

1. Value of pair ${}_{13}C_1$ outcomes
2. (Different) value of triple ${}_{12}C_1$
3. Suits of the 2 pair cards ${}_4C_2$
4. Suits of the 3 triple cards

1. Value of pair ${}_{13}C_1$ outcomes
2. (Different) value of triple ${}_{12}C_1$
3. Suits of the 2 pair cards ${}_4C_2$
4. Suits of the 3 triple cards ${}_4C_3$

1. Value of pair ${}_{13}C_1$ outcomes
2. (Different) value of triple ${}_{12}C_1$
3. Suits of the 2 pair cards ${}_{4}C_2$
4. Suits of the 3 triple cards ${}_{4}C_3$

$$ {}_{13}C_1 \times {}_{12}C_1 \times {}_{4}C_2 \times {}_{4}C_3 = \cdots $$

So how many full house hands are there?

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$$ {}_{13}C_1 \;\times\; {}_{12}C_1 \;\times\; {}_{4}C_2 \;\times\; {}_{4}C_3 = $$
$$ 13 \times 12 \times 6 \times 4 = 3744 $$

Example

How many “pair” hands are there?

How to count?

How to count?

1. Value of pair

How to count?

1. Value of pair
2. Suits of those two cards

How to count?

1. Value of pair
2. Suits of those two cards
3. Values of the other three cards (must be distinct!)

How to count?

1. Value of pair
2. Suits of those two cards
3. Values of the other three cards (must be distinct!)
4. Suits of the other three cards

How to count?

1. Value of pair
2. Suits of those two cards
3. Values of the other three cards (must be distinct!)
4. Suits of the other three cards

Independent!

1. Value of pair
2. Suits of those two cards
3. Values of the other three cards (must be distinct!)
4. Suits of the other three cards

1. Value of pair ${}_{13}C_1$
2. Suits of those two cards
3. Values of the other three cards (must be distinct!)
4. Suits of the other three cards

1. Value of pair ${}_{13}C_1$
2. Suits of those two cards ${}_{4}C_2$
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1. Value of pair ${}_{13}C_1$
2. Suits of those two cards ${}_{4}C_2$
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4. Suits of the other three cards

1. Value of pair ${}_{13}C_1$
2. Suits of those two cards ${}_4C_2$
3. Values of the other three cards (must be distinct!) ${}_{12}C_3$
4. Suits of the other three cards $({}_4C_1)^3$

1. Value of pair ${}_{13}C_1$
2. Suits of those two cards ${}_{4}C_2$
3. Values of the other three cards (must be distinct!) ${}_{12}C_3$
4. Suits of the other three cards $({}_{4}C_1)^3$

$$ {}_{13}C_1 \times {}_{4}C_2 \times {}_{12}C_3 \times ({}_{4}C_1)^3 = \cdots $$

So, the number of “pair” poker hands is:

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$$ {}_{13}C_1 \ \times \ {}_{4}C_2 \ \times \ {}_{12}C_3 \ \times \ ({}_{4}C_1)^3 = $$
$$ 13 \times 6 \times 220 \times 4^3 = 1,098,240 $$

Next time: Section 2.3: Probability