Today's plan:

Section 2.4.: Different values - Equal rights
Section 2.4.1: The continuous case: Cake cutting.

Section 2.4: Different Values - Equal Rights

- Before: everyone agreed on the value of the items to be divided

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- Now: different people assign different values to the items to be divided.

Now the continuous problems are non-trivial, and the discrete problems are sometimes unmanageable.

Section 2.4.1.: The continuous case -Cake Cutting-

Solution

The **two-parties**, *different-values-equal-rights continuous problems are straightforward:*

Solution

"You-cut-I-choose"

- *One person cuts the item into what he considers equal-value parts*
- *The other person chooses a piece*

Example

Ann and Bob buy a $12 cake

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- half Mint

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- half Strawberry

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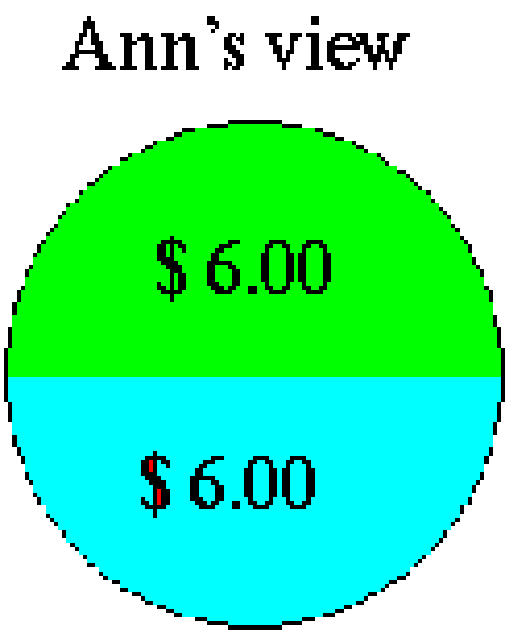
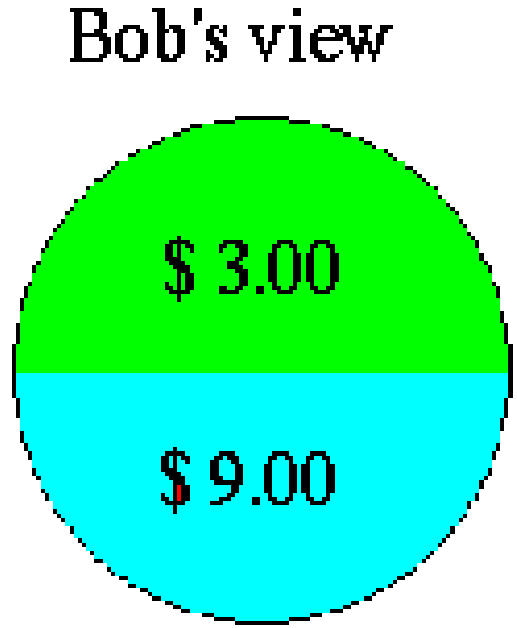
They want to use the you-cut-I-choose method to share the cake.

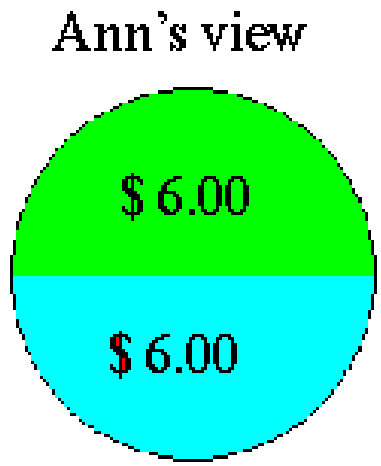
- Ann likes mint and strawberry about the same

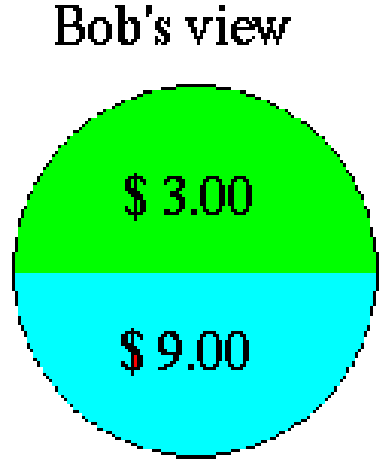
- Ann likes mint and strawberry about the same
- Bob likes strawberry three times more than mint

Let's figure out what the $12 cake is worth to each of the parties.

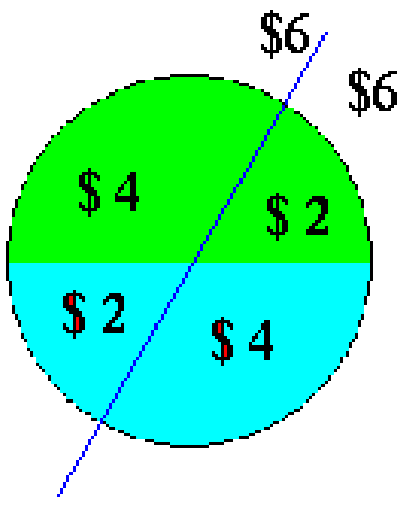




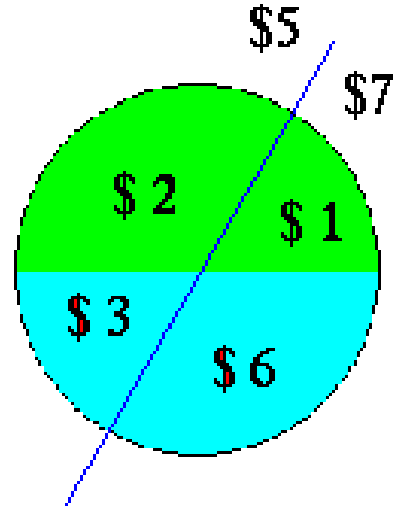




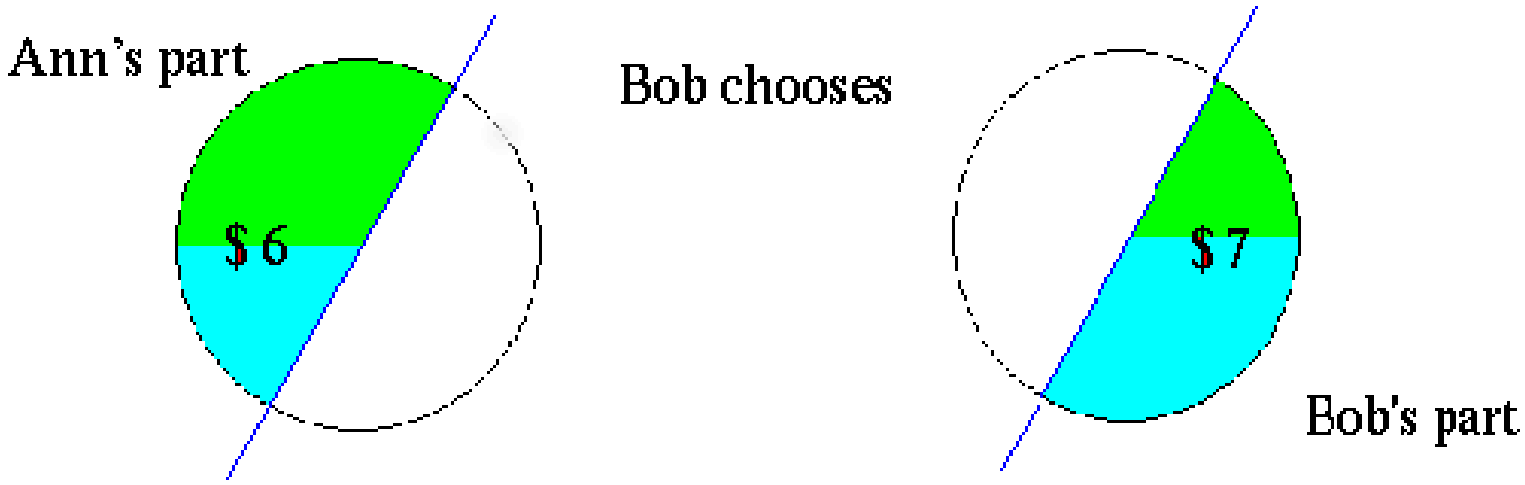
They decide by flipping a coin that Ann cuts the cake.



Ann cuts cake



Now Bob chooses.



- The division is fair since each has a piece they view as worth \$6 or more.

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- It's also envy-free, since each views the other's piece as having equal or lesser value.

Remarks:

- It's to the cutter's advantage to cut it into exactly equal-value parts. Then she's guaranteed a fair share, namely exactly 50% of the value of the item.

- The chooser gets to choose whichever part he considers worth more. So he will end up with at least 50% of the value of the item.

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- There is a slight advantage for the chooser, but the division is still fair.

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- Cut and Choose method

What if we have more than two people dividing the cake? Two methods:

- Cut and Choose method
- Claim and Challenge method

Cut and Choose Method

Example

Three friends, Ann, Bob, and Chad, each contribute $4 to buy a $12 Vanilla-Chocolate cake.

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Example

Three friends, Ann, Bob, and Chad, each contribute $4 to buy a $12 Vanilla-Chocolate cake.

- Ann likes vanilla twice as much as chocolate.
- Bob likes vanilla and chocolate about the same.
- Chad likes chocolate three times as much as vanilla.

Let's figure out how each person values each half. Start with Ann.

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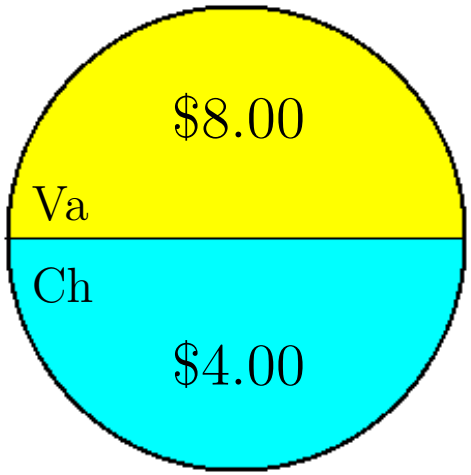
- Call "x" how much Ann values the chocolate half,
- so she values the vanilla half at $2x$.
- The whole cake is worth $x + 2x = 3x = \$12$,
- so $x = \$4$

Let's figure out how each person values each half. Start with Ann.

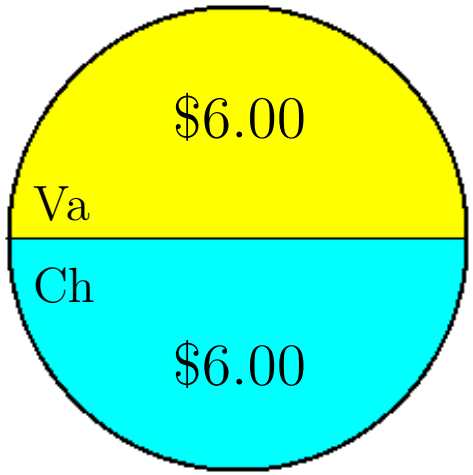
- Call "x" how much Ann values the chocolate half,
- so she values the vanilla half at $2x$.
- The whole cake is worth $x + 2x = 3x = \$12$,
- so $x = \$4$

Ann values the chocolate half at \$4 and the vanilla half at \$8.

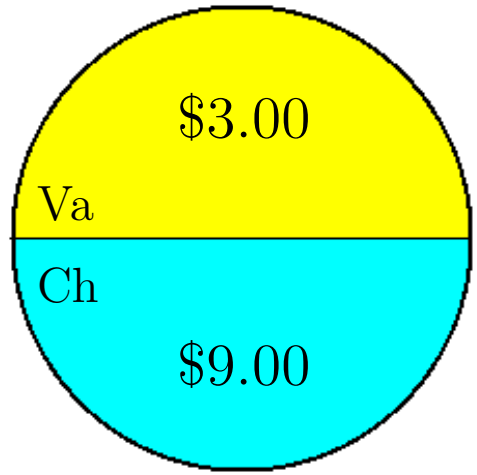
- Ann likes vanilla twice as much as chocolate
- Bob likes vanilla and chocolate about the same
- Chad likes chocolate three times as much as vanilla



Ann



Bob



Chad

Solution

1. *Ann & Bob divide the cake in half with the you-cut-I-choose method.*

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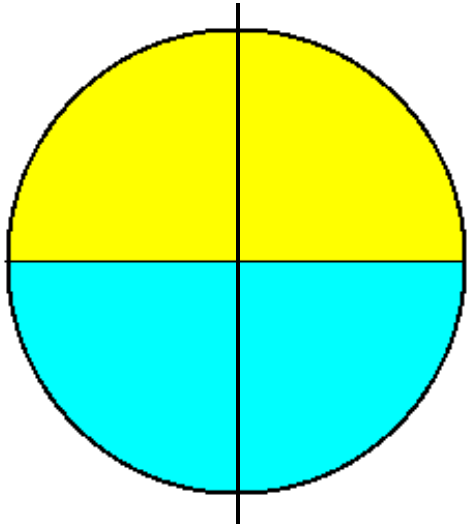
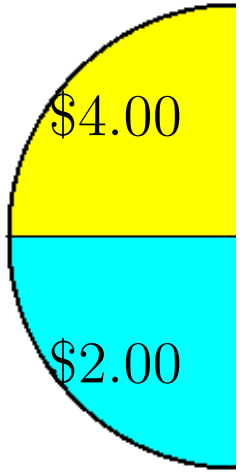
2. *Each of them now cuts his/her half into three equal pieces.*
3. *Chad chooses one third from each half. (Read that carefully!)*

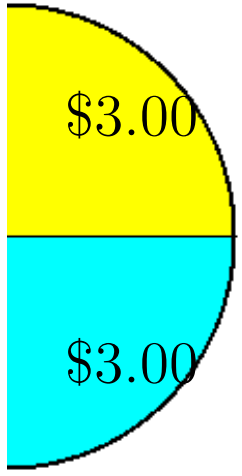
Bob and Ann divide first. Bob cuts, and Ann chooses.



Bob's cut

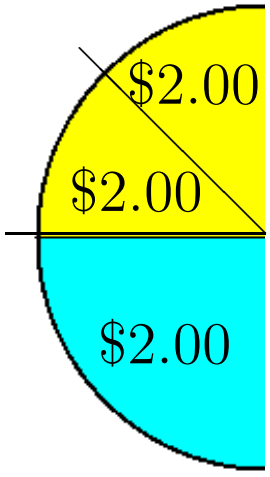


Ann's half

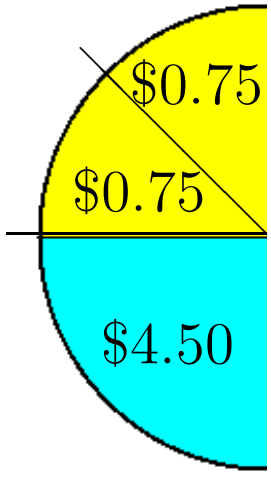


Bob's half

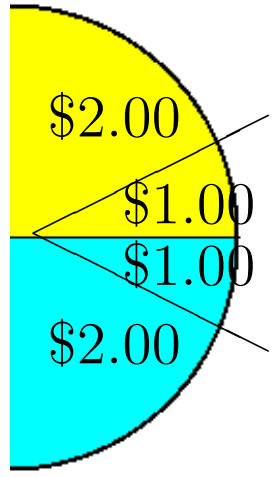
Now Bob and Ann cut their respective halves into what they each consider three equal-valued pieces. Let's also see Chad's view of everything.



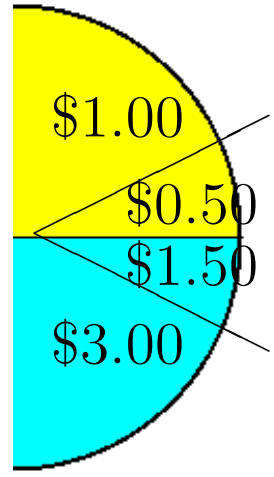
Ann's half



Chad's view



Bob's half



Chad's view

Finally, Chad comes in and chooses.

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- From Ann's half, he chooses the lower part, which to him is worth \$4.50, more than either of the other two parts.

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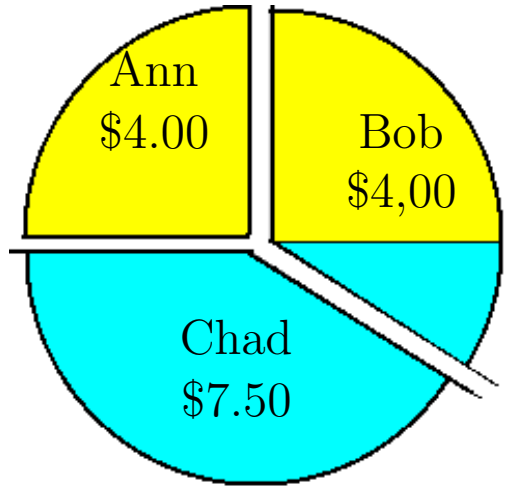
- From Ann's half, he chooses the lower part, which to him is worth \$4.50, more than either of the other two parts.
- From Bob's half, he chooses the lower part, which to him is worth \$3.00, more than either of the other two parts.

The final division:



Remarks:

- Each of the parties ended up with what she/he considers a fair piece ($4.00). So this is a fair division.

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- Each of the parties ended up with what she/he considers a fair piece ($4.00). So this is a fair division.
- But it's not envy-free. In Ann's view, Bob's share is worth $4.67; that is more than her share (and so more than she thinks he deserved).

Remarks:

- To an outsider like us, who knows everyone's values, it seems Chad got the best deal. However, to Ann, Chad's share is worth less than hers.

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- To an outsider like us, who knows everyone's values, it seems Chad got the best deal. However, to Ann, Chad's share is worth less than hers.
- BTW the cuts made are not unique; other scenarios were possible.

Cut and Choose Method

- In the first round, the first two players use the *You-cut-I-choose* method to divide the cake into two halves. The first player cuts, and the second chooses.
- In the second round, each of the two players cuts his/her half into three equal valued pieces.
- The third player chooses one piece from each of the other players parts.

Next time: 2.4.1 continued:
Cut and Choose with more than 3 people, and the Claim and Challenge Method