Content of the course:

1. Voting Systems
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2. Fair Division
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3. Counting and Probability (hard)
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1. Voting Systems
2. Fair Division
3. Counting and Probability (hard)
4. Statistics
Anything you need to know will be on the course website

http://www.math.binghamton.edu/zaremsky/15S_Action.html

(If you don’t want to type that out, just google “Matt Zaremsky” and you’ll find my page pretty quickly.)
Chapter I

Voting Systems
Voting seems straightforward:
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- Candidates run,
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- Candidates run,
- people vote,
Voting seems straightforward:

- Candidates run,
- people vote,
- votes are counted,
Voting seems straightforward:

- Candidates run,
- people vote,
- votes are counted,
- whoever has the majority wins.
But what if
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- we have more than two candidates
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- we have more than two candidates
- different voters get different numbers of votes
A two-candidate election is one with only 2 candidates.
**Definition**

A **two-candidate election** is one with only 2 candidates.

**Example**

With a few exceptions, U.S. presidential elections are (for our purposes) two-candidate elections.
Example

But sometimes there are more than 2 serious candidates.
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But sometimes there are more than 2 serious candidates. In 1968, George Wallace got 46 electoral votes, about 1/4 as many as Humphrey and 1/6 as many as Nixon. Not too shabby! (Except Wallace ran on a platform of racism and segregation so actually it’s kind of embarrassing he did so well in the election.)
Most governmental elections are one-person-one-vote.
Most governmental elections are one-person-one-vote. However, in the private sector often different people have different numbers of votes.
Example

- Stockholders often get a number of votes proportional to the number of stocks they own.
Example

- Stockholders often get a number of votes proportional to the number of stocks they own.
- In a law firm, senior partners might get two votes, whereas junior partners get one.
Our study of voting methods will be broken into 4 categories:
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- One Person – One Vote; Two Candidates
Our study of voting methods will be broken into 4 categories:

- One Person – One Vote; Two Candidates
- One Person – One Vote; Multiple Candidates
- One Person – Multiple Votes; Two Candidates
- One Person – Multiple Votes; Multiple Candidates
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One Person – One Vote; Two Candidates
One Person – One Vote; Two Candidates

- This is ridiculously easy
One Person – One Vote; Two Candidates

- This is ridiculously easy
- The candidate with the largest number of votes is the winner.
The only problem is a tie.
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The only problem is a tie. To simplify things, we’ll just pretend ties never happen. (If Obama and Romney had tied, we probably would have gotten President Romney and VP Biden. Awkward.)
The Math Club is holding an election for president.

There are four candidates: A, B, C, and D.

Of the 20 members in the club...
One Person – One Vote; Multiple Candidates

Example

- The Math Club is holding an election for president.
One Person – One Vote; Multiple Candidates

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One Person – One Vote; Multiple Candidates

Example

- The Math Club is holding an election for president.
- There are four candidates: A, B, C and D.
- Of the 20 members in the club
Example

- 8 vote for candidate A
- 5 vote for candidate B,
- 7 vote for candidate C.
- no votes for D.
A is the candidate with the largest number of votes
A is the candidate with the largest number of votes but that number is less than half of the votes.
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One could interpret this as saying that 8 people support A and 12 people oppose A.
In fact, some of the 5 people voting for B may regret having done so, and wish they had voted for C.
Definition (plurality method)

Under the plurality method the candidate with the largest number of votes is declared the winner.
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Using the plurality method A is a winner!
Remark

The winning candidate may or may not have a majority of votes.
Remark

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Definition

A majority is more than half of the votes.
Remark

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Definition

A majority is more than half of the votes.

In the Math Club election a majority requires at least 11 votes, but no one has a majority.
Drawback of the plurality method

Voters only get to express their first choice. Maybe all 5 people who voted for B would prefer C over A.
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Advantages of the plurality method

If a candidate happens to get a majority, then that candidate will be the winner under the plurality method. We say that the plurality method respects the majority criterion.
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Majority Criterion

**IF** a candidate receives a majority of first-place votes, **THEN** that candidate ought to be the winner of the election.
Remark

The majority criterion does not say that a candidate must have a majority in order to win.
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Some methods, like the plurality method respect the majority criterion, but other methods don’t.
In many instances the majority criterion does not even apply because none of the candidates have a majority.
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Example (Mathclub elections)

- Of the 20 members in the club
  - 8 vote for candidate A
  - 5 vote for candidate B,
  - 7 vote for candidate C.
  - no votes for D.
We need 11 votes for the majority.
We need 11 votes for the majority. No one of candidates has the majority. BUT
We need 11 votes for the majority. No one of candidates has the majority. BUT

Remark

*The majority criterion is NOT violated here! (Since no one even had a majority.*)*
Remarks:

- The plurality method, despite its drawbacks, is widely used.
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- The plurality method, despite its drawbacks, is widely used.
- In the U.S. presidential election, each state uses its popular vote to decide how to allocate the electoral votes in the electoral college. Most states use the plurality method.
Example

In the 2000 presidential election

- Gore carried Wisconsin with only 47.89% of the vote, to Bush’s 47.68%.
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Example

In the 2000 presidential election

- Gore carried Wisconsin with only 47.89% of the vote, to Bush’s 47.68%.
- Bush carried Nevada with 49.52% of the vote, to Gore’s 45.97%.

In both cases no candidate had a majority.