

Content of the course:

1. Voting Systems

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2. Fair Division

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3. Counting and Probability (hard)

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3. Counting and Probability (hard)
4. Statistics

Anything you need to know will be on the course website

http://www.math.binghamton.edu/zaremsky/15S_Action.html

(If you don't want to type that out, just google "Matt Zaremsky" and you'll find my page pretty quickly.)

Chapter I

Voting Systems

Voting seems straightforward:

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- ▶ Candidates run,
- ▶ people vote,
- ▶ votes are counted,
- ▶ whoever has the majority wins.

But what if

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- ▶ we have more than two candidates

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- ▶ we have more than two candidates
- ▶ different voters get different numbers of votes

Definition

A **two-candidate election** is one with only 2 candidates.

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Example

With a few exceptions, U.S. presidential elections are (for our purposes) two-candidate elections.

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In 1968, George Wallace got 46 electoral votes, about $1/4$ as many as Humphrey and $1/6$ as many as Nixon. Not too shabby! (Except Wallace ran on a platform of racism and segregation so actually it's kind of embarrassing he did so well in the election.)

Most governmental elections are
one-person-one-vote.

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However, in the private sector often different people have different numbers of votes.

Example

- ▶ Stockholders often get a number of votes proportional to the number of stocks they own.

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- ▶ In a law firm, senior partners might get two votes, whereas junior partners get one.

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- ▶ One Person – Multiple Votes; Two Candidates
- ▶ One Person – Multiple Vote; Multiple Candidates

One Person – One Vote; Two Candidates

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- ▶ This is ridiculously easy

One Person – One Vote; Two Candidates

- ▶ This is ridiculously easy
- ▶ The candidate with the largest number of votes is the winner.

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(If Obama and Romney had tied, we probably would have gotten President Romney and VP Biden. Awkward.)

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Example

- ▶ The Math Club is holding an election for president.

One Person – One Vote; Multiple Candidates

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- ▶ There are four candidates: **A**, **B**, **C** and **D**.

One Person – One Vote; Multiple Candidates

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- ▶ The Math Club is holding an election for president.
- ▶ There are four candidates: **A**, **B**, **C** and **D**.
- ▶ Of the 20 members in the club

Example

- ▶ 8 vote for candidate **A**
- ▶ 5 vote for candidate **B**,
- ▶ 7 vote for candidate **C**.
- ▶ no votes for **D**.

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- ▶ but that number is less than half of the votes.
- ▶ One could interpret this as saying that 8 people support **A** and 12 people oppose **A**.

- ▶ In fact, some of the 5 people voting for **B** may regret having done so, and wish they had voted for **C**.

Definition (**plurality method**)

Under the **plurality method** the candidate with the largest number of votes is declared the winner.

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Using the plurality method **A** is a winner!

Remark

The winning candidate may or may not have a majority of votes.

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Definition

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Definition

A **majority** is more than half of the votes.

In the Math Club election a majority requires at least 11 votes, but no one has a majority.

Drawback of the plurality method

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Maybe all 5 people who voted for **B** would prefer **C** over **A**.

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We say that the plurality method respects the **majority criterion**.

Majority Criterion

IF a candidate receives a majority of first-place votes,
THEN that candidate ought to be the winner of the election

Remark

*The majority criterion **does not say** that a candidate **must** have a majority in order to win.*

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Some methods, like the plurality method respect the majority criterion, but other methods don't.

In many instances the majority criterion does not even apply because none of the candidates have a majority.

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Example (Mathclub elections)

- ▶ Of the 20 members in the club
 - ▶ 8 vote for candidate **A**
 - ▶ 5 vote for candidate **B**,
 - ▶ 7 vote for candidate **C**.
 - ▶ no votes for **D**.

We need 11 votes for the majority.

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No one of candidates has the
majority. BUT

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Remark

*The majority criterion is **NOT**
violated here! (Since no one even had
a majority.)*

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- ▶ In the U.S. presidential election, each state uses its popular vote to decide how to allocate the electoral votes in the electoral college. Most states use the plurality method.

Example

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- ▶ Gore carried Wisconsin with only 47.89% of the vote, to Bush's 47.68%.

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- ▶ Bush carried Nevada with 49.52% of the vote, to Gore's 45.97%.

In both cases no candidate had a majority.