Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Explain your answers and reasoning.

Name: ______________________________

ID number: ______________________________

Name that you would like to be called: ______________________________

Instructions:

- Read problems very carefully. If you have any questions raise your hand.
- The correct final answer alone is not sufficient for full credit - try to explain your answers as much as you can (except for Problem 1). This way you are minimizing the chance of losing points for not explaining details, and maximizing the chance of getting partial credit if you fail to solve the problem completely. However, try to save enough time to seriously attempt to solve all the problems.
- You final answers need to be simplified only if this is required in the statement of the problem. Otherwise, you can leave them in any form you wish. Your final answer should not involve infinite sums.
- You are allowed to use only the items necessary for writing. Notes, books, sheets, calculators, phones are not allowed, please put them away. If you need more paper please raise your hand (you can write on the back pages).
- There is no need to simplify numerical quantities like $4 \cdot 0.3 \cdot \frac{0.3+\frac{3}{4}}{\frac{7}{5}+\frac{23}{547}}$ or to reduce fractions like $\frac{3}{6}$ to lowest terms. There is also no need to simplify factorials such as 5! or binomial coefficients such as $\binom{7}{3}$. Your final answer should not involve infinite sums.

Question:	1	2	3	4	5	6	7	Total
Points:	15	10	10	10	10	10	10	75
Score:								

1. (15 points) (a) Let A and B be events such that $\mathbb{P}(A) = 1/8$ and $\mathbb{P}(B) = 1/4$. What possible values can $\mathbb{P}(A|B)$ have?

   (b) Let C and D be independent events such that $\mathbb{P}(C) = 1/4$ and $\mathbb{P}(D) = 1/3$ and $\mathbb{P}(C \cup D) = 1/2$. Are C and D independent?

   (c) Let X be a random variable with probability distribution function $p_X(0) = 1/6, p_X(1) = 1/3, p_X(2) = 1/2$. What are $\mathbb{E}[X]$ and $\mathbb{E}[(X+1)^{-1}]$ (you do not need to simplify)?

   (d) Let Y be a random variable that equals 1 with probability p and 0 with probability $1-p$. Compute the moment generating function of Y.

2. (10 points) You roll a fair 4 sided die twice. The faces of the die are labelled $1, 2, 3, 4$. Assume the dice rolls are independent. You keep track of the order of the dice rolls.

   (a) List the sample space of all possible outcomes.

   (b) What is the probability that any of the simple events occur?

   (c) What is the probability the first dice roll is greater than the second?

   (d) What is the probability the sum of the two dice rolls is 6?

   (e) What is the probability the sum of the two dice rolls is 9?

3. (10 points) You are waiting at a subway stop and decide to take subway 1, 2 or 3, whichever line arrives first. Subway 1 has a 50 percent chance of being next, subway 2 and subway 3 each have a 25 percent chance of being next. Line 1 has a 70 percent chance of not being delayed.
   Line 2 has a 30 percent chance of not being delayed.
   Line 3 has a 50 percent chance of not being delayed.

   Given that your train was not delayed, what is the probability you took subway 2?

4. (10 points) You are dealt 5 cards are of a standard deck of 52.

   (a) What is the probability the first two cards you are dealt are aces?

   (b) What is the probability exactly 3 of your 5 cards are spades?

5. (10 points) During an at-bat, a given baseball player hits a single with probability $3/10$, a double with probability $1/10$ and makes an out the rest of the time.

   After a single they score a run with probability $1/8$ and don't with probability $7/8$.

   After a double they score a run with probability $1/4$ and don't with probability $3/4$.

   After an out they can't score a run.

   What is the expectation and variance of the number of runs they score in an at-bat?

6. (10 points) Person A and Person B play a game where they take turns flipping an unfair coin. The coin has $1/4$ chance of being heads and is tails otherwise.

   The first person to flip a heads wins.

   If person A wins, what is the probability they won on their third coin flip (the fifth flip in total)?

7. (10 points) A security system consists of 10 sensors. Assume that when there is an intruder each sensor is activated with probability .9, independently. The security system sends an alarm if at least 3 sensors are activated, and it only sends at alarm when at least 3 sensors are activated.

   (a) When there is an intruder, what is the probability the security system sends an alarm?

   (b) Given that the system sends an alarm, what is the probability exactly 7 sensors were activated?