

Homework 5 - Due Friday, Feb. 28

Do Problems 9.2, 9.3, 9.7, 9.15, 9.19, 9.23, 9.29 and the problem below.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. An experimental trial produces random variables X and Y with covariance $\sigma_{XY} = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$. To estimate σ_{XY} , we perform N independent trials and form the estimator

$$\widehat{R}_N = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})$$

where X_1, \dots, X_N and Y_1, \dots, Y_N are random samples from a population with the same distribution as X and Y , respectively. As usual $\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i$ and $\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$.

Show that if $Var[X_i Y_i]$, $Var[Y_i]$, $Var[X_i]$ is finite, then \widehat{R}_N is an unbiased, consistent estimator of σ_{XY} .

Hint: Why can you assume without loss of generality that $\mathbb{E}[X] = \mathbb{E}[Y] = 0$?