

Homework 6

Do the problems on webwork and turn the following problems in class on Mar. 15st.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. Which of the following functions can be a probability density function for some continuous random variable. Explain your reasoning.

1)

$$f(t) = \begin{cases} \pi \cos(\pi t), & \text{for } 0 \leq t \leq 5/2 \\ 0, & \text{otherwise.} \end{cases}$$

2)

$$g(t) = \begin{cases} e^{-t}, & \text{for } t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

3)

$$h(t) = \begin{cases} \frac{1}{e-1} e^{\sin(t)} \cos(t), & \text{for } 0 \leq t \leq \pi/2 \\ 0 & \text{otherwise.} \end{cases}$$

4)

$$j(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1, & \text{for } 0 \leq t \leq 1/2 \\ 0, & \text{for } 1/2 < t < 1 \\ 1, & \text{for } 1 \leq t \leq 3/2 \\ 0, & \text{for } 3/2 < t. \end{cases}$$

5)

$$k(t) = 1 \text{ for all } t \in \mathbb{R}$$

Problem 2. (a) You roll a fair 6-sided dice, let Y be the outcome of the dice roll. Then conditioned on the event $\{Y = k\}$ for $k = 1, \dots, 6$ you randomly choose, X , to be uniformly distributed between 0 and k .

- Use the law of total probability to compute $\mathbb{P}(\{X < x\})$.
- Use part a) to compute $f_X(x)$.
- What is the expectation of X .