

Homework 5

Do the problems on webwork and turn the following problems in class on Mar. 1st.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. An insect lays a random number of eggs. The number of eggs has a Poisson distribution with mean 50. Each egg develops into an offspring with probability $1/4$.

Let Y be the number of eggs laid and X be the number of offspring.

- Conditioned on the event that $\{Y = n\}$, what is the pmf of X ? (Your answer should involve n).
- Use the law of total probability to show X is also a Poisson random variable. What is its mean? (You need to compute the probability $X = k$ for any k and simplify the infinite sum).
- What is $\mathbb{E}[2X^2 + X - 1]$?

Problem 2. (a) Compute the moment generating functions of the random variable, X , with the following pmf:

$$p_X(x) = \begin{cases} 1/4 & x = 1 \\ 3/4 & x = -2 \\ 0 & \text{otherwise} \end{cases}$$

- Compute the expectation and variance of a random variable with the following moment-generating function

$$m_Y(t) = 1/3e^{-t} + \frac{1}{6}e^{4t} + 1/2e^{9t}$$

Problem 3. Find the distributions of the random variables that have each of the following moment-generating functions:

(a) $m_X(t) = [(1/4)e^t + (3/4)]^3$

(b) $m_Y(t) = e^{3(e^t - 1)}$

(c) $m_Z(t) = \frac{1/4e^t}{1 - 3/4e^t}$