

Homework 12

Do the problems on **Webwork** and turn the following problems in class on May 10th.

Homework should be written neatly and clearly explained. If it requires more than one sheet, the sheets must be stapled. Include your name and id number in the top right corner of your homework.

Problem 1. Let X_1, \dots, X_n be independent, identically distributed random variables with probability density function

$$f(x) = \begin{cases} 3\alpha^3 x^{-4}, & \text{if } x > \alpha, \\ 0, & \text{otherwise} \end{cases}$$

where α is a constant greater than 0.

Let $X_{(1)} = \min\{X_1, \dots, X_n\}$.

Compute the probability density function for $X_{(1)}$ and $\mathbb{E}[X_{(1)}]$.

Problem 2. Fifty athletes is trying to qualify for the finals. Each of them has to pass the preliminary stage and the advanced stage. Each athlete passes the preliminary stage with the probability 0.5 and the advanced stage with probability 0.3 independently. Each athlete passes stages independently of all other athletes.

Let N be the number of people who pass the preliminary stage and fail to pass the advanced stage. Use the Central Limit Theorem to estimate the probability $\mathbb{P}(N \leq 20)$.