For Test 2 – Some other items

What is meant by marginal cost?

This question makes the most sense when put in terms of a number of units being produced, say $x = 16$.

So, the "marginal cost at a level of production of 16 units" means the cost of producing the 17th item.

The way to find this cost exactly is to compute $C(17) - C(16)$ (it's a margin of cost). But to do this for each value of x you might be interested in is tedious. The short cut way to find the marginal cost approximately is to find an expression for $\frac{C(x+1) - C(x)}{x+1-x}$

That expression is of course $C'(x)$.

$$C'(x) \approx \frac{C(x+1) - C(x)}{\cancel{x}+1-\cancel{x}}$$

or $C'(x) \approx C(x+1) - C(x)$

<u>Ex</u> Given a cost fcn. $C(x) = 80\sqrt{x} + 950$

what is the marginal cost at a level of production $x = 100$? (Assume cost is in \$)

$$C'(x) = 80 \cdot \frac{1}{2} x^{-1/2} = \frac{40}{\sqrt{x}}$$

$$C'(100) = \frac{40}{\sqrt{100}} = \frac{40}{10} = \$4$$

What this means is that the cost of producing the 101st item is \$4

Generally, we need to find dy/dx given an equation in x and y.

<u>Ex</u> $3xy - \frac{y}{3} = \frac{2}{x}$

Differentiate term by term.

$$3xy - \frac{y}{3} = \frac{2}{x}$$

straight differentiation w.r.t x

Needs prod. rule + implicit

just implicit

$$3\left(1\cdot y + x\cdot \frac{dy}{dx}\right) - \frac{1}{3}\frac{dy}{dx} = \frac{-2}{x^2}$$

(right side from $2x^{-1}$)

$$3y + x\frac{dy}{dx} - \frac{1}{3}\frac{dy}{dx} = \frac{-2}{x^2}$$

$$\frac{dy}{dx}\left(x - \frac{1}{3}\right) = \frac{-2}{x^2} - 3y$$

$$\boxed{\frac{dy}{dx} = \frac{-2/x^2 - 3y}{x - 1/3}}$$

What is the marginal cost at a level of production of $x = 200$?

$$C'(200) = \frac{40}{\sqrt{200}} = \frac{40}{10\sqrt{2}} \approx \frac{4}{1.414}$$

or about $4/1.414 or $2.82 to produce the 201st unit

Notice how marginal cost went down as production rose?

Implicit differentiation with a product rule and an e^u: In this problem there is chain rule and product rule and derivative of e^u

$f = e^{xy^2}$ find $f'(x)$

Soln $e^u = e^{x \cdot y^2}$, $\frac{du}{dx} = 1 \cdot y^2 + x \cdot 2y \frac{dy}{dx}$

$$f'(x) = e^u \frac{du}{dx} = e^{xy^2} \cdot 2xy \frac{dy}{dx}$$