1

Ex 10.7 $C'(x) \approx C(x+1) - C(x)$

p. 93 #15 $p'(q) \approx p(q+1) - p(q)$

Price might go up or down if one more item is sold.

#5 $R'(x) \approx R(x+1) - R(x)$

Revenue increase or decrease for $(x+1)$th item sold

p. 99 #16 $p(x) = 100 + \dfrac{50}{\ln x}, \quad x>1$

~~$p'(x)$~~

Find $R'(x)$. – First we need $R(x)$.

Revenue = (price per item)(# of items sold)

$$R(x) = p \cdot x = \left(100 + \frac{50}{\ln x}\right)x$$

$$= 100x + \frac{50x}{\ln x}$$

$$R'(x) = 100 + \frac{(50)\ln x - (50x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$R'(x) = 100 + \frac{50\ln x}{(\ln x)^2} - \frac{50}{(\ln x)^2}$$

$$= \boxed{100 + \frac{50}{\ln x} - \frac{50}{(\ln x)^2}}$$

p.98 #7 I assigned this one because it gives ~~cost~~ $C(q)$ but asks for $R(p)$ and $P(p)$. So, you will have to rewrite $C(q)$ as a fcn. of p, since $P(p) = R(p) - C(p)$

Given $C(q) = 200q + 35{,}000$

$q(p) = 1500 - 1.5p$

a) $R(p) = \frac{\text{price}}{\text{unit}} \cdot \#\text{ units sold}$

$= p \cdot q(p)$

$= p(1500 - 1.5p)$

$R(p) = 1500p - 1.5p^2$

b) Find $P(p)$; that is, $R(p) - C(p)$

$C(q) = 200q + 35{,}000$

by subsitution $C(p) = 200(1500 - 1.5p) + 35{,}000$

$= 300{,}000 - 300p + 35{,}000$

$= 335{,}000 - 300p$

So $P(p) = R(p) - C(p)$

$= 1500p - 1.5p^2 - 335{,}000 + 300p$

$= 1800p - 1.5p^2 - 335{,}000$

$P(p) = -1.5p^2 + 1800p - 335{,}000$

~~bad~~

c) $P'(p) = -3p + 1800$ d) $P'(500) = \$300$

p. 98 # 7d con'd

$P'(500) = \$300$ means the profit for raising the price by \$1 over \$500 (\$501) is \$300, but if you keep raising the price the profit for the next unit sold will start to go down, and at some value of p,

$P'(p) = 0$ (that is the topic of the next unit of this course)

p. 92 # 5 This problem also gives a fcn. in terms of one variable (p) and asks for another fcn. related to it in terms of the other variable (q)

Given: $q(p) = 400 - 100p$ (i.e. $q = 400 - 100p$)

Find $R'(q)$. First you need $R(q)$

Revenue = price × quantity

$R(q) = \underbrace{p(q)} \times q$

But we're given $q(p)$, so rearrange it to be $p(q)$

$q = 400 - 100p \longrightarrow p = \frac{q}{100} - \frac{400}{100} = \frac{q}{100} - 4$

#5 con'd p.92

So $R(q) = q \cdot p(q) = q\left(\frac{q}{100} - 4\right)$

$$R(q) = \frac{q^2}{100} - 4q$$

Then $R'(q) = \frac{2q}{100} - 4 = \frac{q}{50} - 4$

a) Find $R'(60)$: $R'(60) = \frac{60}{50} - 4 = -\$2.80$

The revenue for the 61st item is negative. You're losing \$.

b) $R'(120) = \frac{120}{50} - 4 = -\$1.60$

Still negative, but ~~pro~~ revenue is starting to rise.

c) $R'(400) = \frac{400}{50} - 4 = 8 - 4 = \$4$

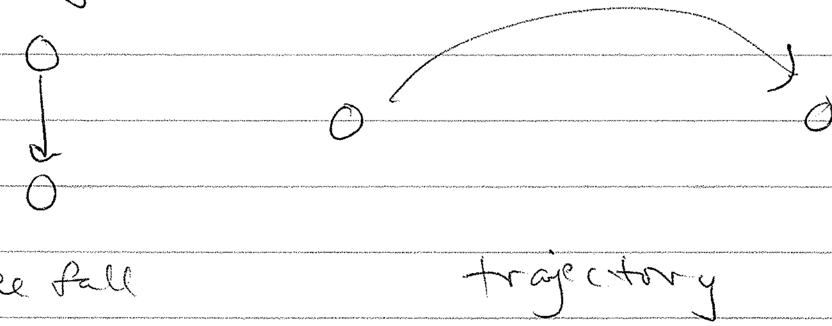
So for the 401st unit, revenue goes up by \$4.

To find how many units you need to sell to see revenue rise, set $R' = 0$. That's the next unit.

p. 107
~~#9~~

$s(t)$ displacement fcn., where s is dist traveled as a fcn. of t

~~Usually~~ Usually we're interested in the effect of gravity on an object in free fall or trajectory motion.



Both are governed by acceleration due to gravity, which is $-32.2\ ft/sec^2$ or $-9.8\ m/sec^2$

(Negative refers to direction)

a) Find $s(2)$ ($t = 2$ sec in free fall)
$s(t) = -16t^2 + 100$ → dist above the ground
$s(2) = -16(4) + 100 = 36$ ft

The reference point of the displacement equation is $s(t) = 0$ ← hits the ground

b) $-16t^2 + 100 = 0 \rightarrow t^2 = \frac{100}{16}$ t^2 $\Rightarrow \frac{10}{4}$ sec

(5)

p 107
#9b

Velocity is the first derivative of displacement.

Hence, $v(t) = s'(t)$

$v(t) = s'(t) = -32t \longrightarrow v(2) = -64 \text{ ft/sec}$

* DO NOT NEGLECT UNITS

Velocity is a vector and negative value means down. Speed is a scalar and so we take $|v(t)| = |-64 \text{ ft/sec}| = 64 \text{ ft/sec}$

c) Acceleration is der. of velocity (i.e. second der. of displacement)

$s''(t) = v'(t) = a(t) = -32 \text{ ft/sec}^2$

downward in direction

~~p. 121 #4~~ $\cancel{R(x) = 50x - .4x^2, \quad R(t) = ?}$

$\cancel{R'(t) = \frac{dR}{dt} = \frac{dR}{dx} \cdot \frac{dx}{dt}}$

See next page

$\cancel{C(x) = 5x + 15}$

$\cancel{C'(t) = \frac{dC}{dx} \cdot \frac{dx}{dt}}$

$\cancel{P(x) = R(x) - C(x)}$

$\cancel{P'(t) = \frac{dP}{dx} \cdot \frac{dx}{dt}}$

p. 98 #6 $q(p) = -\frac{4(p+1)^2}{3} + 80$

$R(p) = p \cdot q(p) = p\left(\underbrace{-\frac{4(p+1)^2}{3} + 80}_{q(p)}\right)$

$R(p) = -\frac{4p(p+1)^2}{3} + 80p$

~~R(p)~~ Revenue = demand × price
(# of items sold)

You need to create this from given demand fcn
– just multiply by p

$R = px$ or pq

$R(p) = -\frac{4p(p+1)^2}{3} + 80p$

$R(p) = -\frac{4}{3}\left(p(p+1)^2\right) + 80p$

$R'(p) = -\frac{4}{3}\left(1 \cdot (p+1)^2 + 2(p+1)^1 \cdot p\right) + 80$

$R'(p) = -\frac{4}{3}\left((p+1)^2 + 2p(p+1)\right) + 80$

$= -\frac{4}{3}\left(p^2 + 2p + 1 + 2p^2 + 2p\right) + 80$

$R'(p) = -\frac{4}{\cancel{3}} \cdot \cancel{3}p^2 - \frac{4}{3} \cdot 4p - \frac{4}{3} + 80$ etc

p. 107 #4c

$g(x) = -10^{3x^2-2}$

$g'(x) = ?$ The form of the given fcn is a^u, which has derivative $a^u \cdot \ln a \cdot du/dx$

Hence $g'(x) = -10^{3x^2-2} \cdot \ln 10 \cdot 6x$

$-a^u \cdot \ln a \cdot du/dx$

Notice that $a \neq -10$ since it isn't given as $(-10)^{3x^2-2}$. Notice also that you cannot combine by multiplication the coeff $\ln 10 \cdot 6$ in the answer with the base 10. Thus,

$$g'(x) = -6x \cdot \ln 10 \cdot 10^{3x^2-2}$$

Now, $g''(x)$ entails the product rule on $x \cdot 10^{3x^2-2}$. Get the coeff $-6 \cdot \ln 10$ out of the way: $g'(x) = -6 \cdot \ln 10 \cdot x \cdot 10^{3x^2-2}$

Then $g''(x) = -6 \cdot \ln 10 \left[1 \cdot 10^{3x^2-2} + \underline{x \cdot 10^{3x^2-2} \cdot \ln 10 \cdot 6x}\right]$

or $g''(x) = -6 \cdot \ln 10 \cdot 10^{3x^2-2}\left[1 + 6x^2 \cdot \ln 10\right]$ final

#7. $f(x) = e^{-2x}$, $f'(x) = -2e^{-2x}$, $f''(x) = (-2)^2 e^{-2x}$

It's best <u>not</u> to write coeffs as
$-2, 4, -8, 16, \ldots$, but as $(-2)^n$

Then $f^{(28)} = (-2)^{28} e^{-2x} = 2^{28} e^{-2x}$

positive, since $(-1)^{\text{even}} = 1$

and $f^{(29)} = (-2)^{29} e^{-2x}$

$= -2^{29} e^{-2x}$

negative, since $(-1)^{\text{odd}} = -1$

My form was $f^{(n)}(x) = (-1)^n 2^n e^{-2x}$

The book's is $f^{(n)}(x) = (-2)^n e^{-2x}$

Mine is inspired by terms given in series
like Taylor series. Don't use
mine if you like the book's.

p.121 #4 Related Rates

Scheme: $\dfrac{d\_}{dt} = \dfrac{d\_}{dx} \cdot \dfrac{dx}{dt}$ ← Given value of rate

↑ Might be dR/dt, dC/dt, dP/dt

↑ you differentiate the given fcn to get this

$\dfrac{d\_}{dt}$ is evaluated at given value of x.

Given: $R(x) = 50x - .4x^2$, $C(x) = 5x + 15$

x = daily production

a) Find: $\dfrac{dR}{dt}$ when $x = 40$ units and $\dfrac{dx}{dt} = 10 \dfrac{\text{units}}{\text{day}}$

↑ You identify this is what you are seeking

↑ Given value

↑ Another given value

$$\left.\frac{dR}{dt}\right|_{x=40} = \left.\frac{dR}{dx}\right|_{x=40} \cdot \frac{dx}{dt} = \underset{\text{at } x=40}{(50 - .8x)}(10)$$

$$= (50 - 32)(10)$$

$$= \$180/\text{day}$$

b) Find $\frac{dC}{dt}$: $\frac{dC}{dt} = \frac{dC}{dx} \cdot \frac{dx}{dt}$

Given $C(x) = 5x+15$: $\frac{dC}{dx} = 5$, $\left.\frac{dC}{dt}\right|_{x=40} = \frac{dC}{dx} \cdot \frac{dx}{dt} = 5 \cdot 10$

$\frac{dC}{dt} = \$50/\text{day}$ ($x=40$ doesn't enter into it — why?)

c) Find $\frac{dP}{dt}$. Need $P(x) = R(x) - C(x)$
Then $P'(x) = R'(x) - C'(x)$

$P'(x) = \frac{dP}{dx} = 50 - .8x - 5 = 45 - .8x$

$\left.\frac{dP}{dt}\right|_{x=40} = \left.\frac{dP}{dx}\right|_{x=40} \cdot \frac{dx}{dt}$

$= \left(45 - .8(40)\right)(10)$

$= (13)(10) = \$130/\text{day}$

p. 121 #2

q thousand watches at p dollars/watch obey the demand fcn:

$$p+q^2=144$$

Find $\frac{dq}{dt}$ when $\frac{dp}{dt}$ = \$2/wk

and $q=9$, $p=63$.

This is solved by the related rate model we talked in general about, given the problem:

1 - Identify the rate you seek (dq/dt)

2 - Identify the givens (dp/dt = \$2, $q=9$, $p=63$)

3 - Find an eqn. for the rate you seek + substitute the values given

This eqn. however is an implicit fcn. q of p. I'm treating it that way rather than as p of q because they give dp/dt, so I'll need the formulation

$$\frac{dq}{dt}=\frac{dq}{dp}\cdot\frac{dp}{dt}$$

p121 #2 con'd. From $p + q^2 = 144$

Find $\frac{dq}{dp}$: $1 + 2q \frac{dq}{dp} = 0$ by implicit

$$\frac{dq}{dp} = \frac{-1}{2q}$$

Set up: $\frac{dq}{dt} = \frac{dq}{dp} \cdot \frac{dp}{dt} = \frac{-1}{2q} \cdot \cancel{\#}2$

Thus, $\left.\frac{dq}{dt}\right|_{q=9 \text{ watches}} = \left.\frac{\cancel{\#}2}{2q}\right|_{q=9} = \boxed{\frac{-\$2}{18}}$

That is, $\frac{dq}{dt} = \$1/9$ of 1000 watches

or, at $p = \$63$,

sales q is decreasing at

a rate of 111 watches/wk

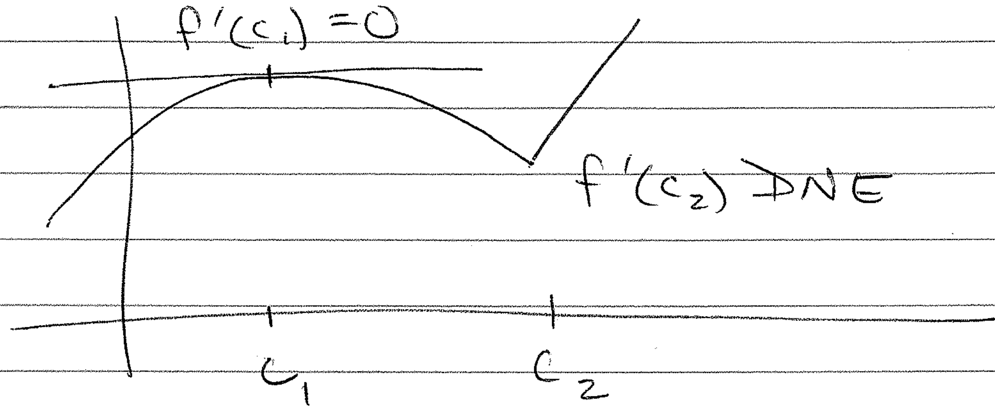
Critical "numbers" c

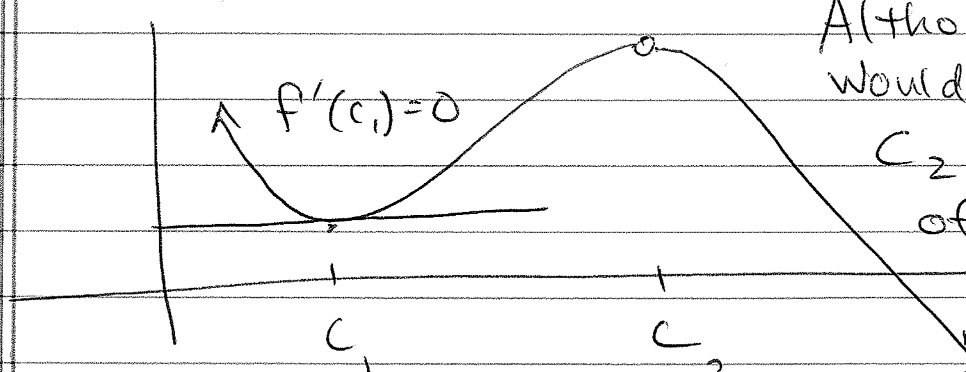
- c such that $f'(c)=0$

  or $f'(c)$ DNE

- Make c itself is in domain

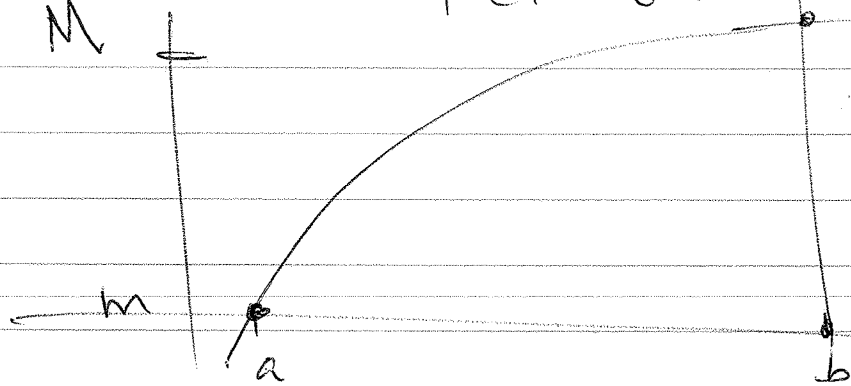
While the book uses "Critical pts" and "values", ~~the~~ "critical number" is a better & more widely used & correct term.



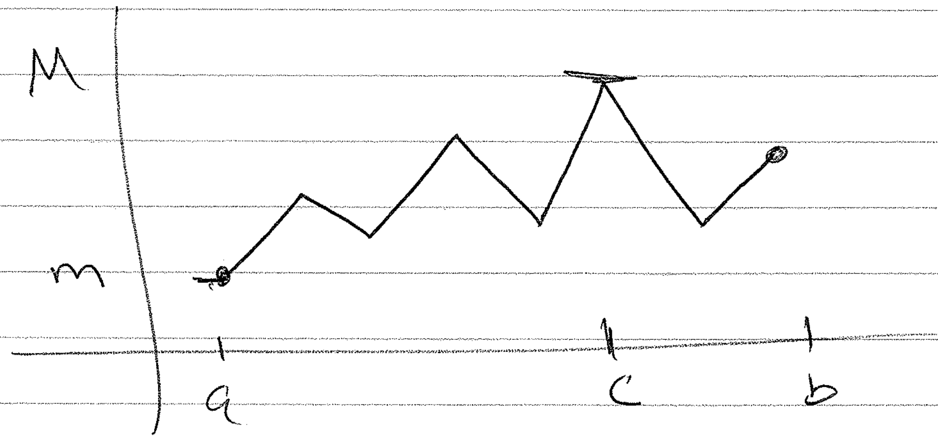


Altho' $f'(c_2)$ would be zero, c_2 is left out of the domain, so it is not a critical number.

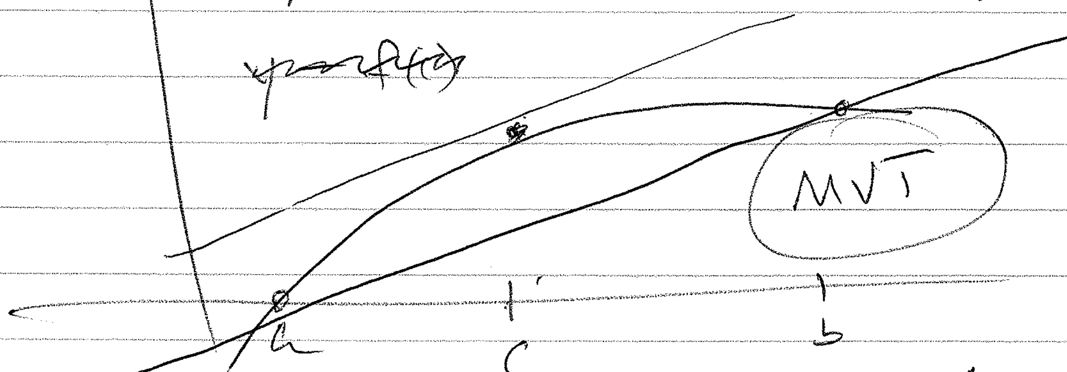
f cts on [a, b]



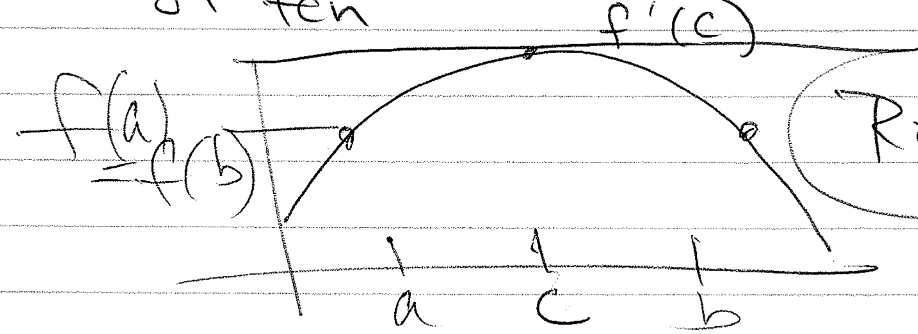
Each has an M (abs max) and m (abs min)



Add f' exists on (a, b)



mean value of fcn $\dfrac{f(b)-f(a)}{b-a} = f'(c)$



Rolle's