Math 220 - Calculus f. Business and Management - Worksheets 5 & 6

Worksheet 5 & 6 - Exponents and Logs

Solving exponential equations

Exercise 1:

Find the value of x in each expression.

$$\textit{1a: } 5^{2x+3} = 5^4, \quad \textit{1b : } 3^{4x-7} = 3^{6-2x}, \quad \textit{1c: } 7^{x^2-8} = 7^{2x},$$
$$\textit{1d: } 4^{x^3} = 4^x, \quad \textit{1e: } 2^x = 4^{x+5}, \quad \textit{1f: } 9^{-4x} - 3^{2x-3} = 0$$

Conversion between exponents and logs

Exercise 1:

Rewrite each exponential expression as a log and each log as an exponent.

$$\textit{1a: } 10^4 = 10,000, \quad \textit{1b: } 2^3 = 8, \quad \textit{1c: } 1/25 = 5^{-2}, \quad \textit{1d: } 9^{1/2} = 3,$$
$$\textit{1e: } \log_3 81 = 4, \quad \textit{1f: } \log_7 49 = 2, \quad \textit{1g: } \log 0.01 = -2, \quad \textit{1h: } \ln e = 1$$

Exercise 2: *Without a calculator, find the value of x in each equation.*

$$\textit{2a: } \log_5 x = 3, \quad \textit{2b: } \log 1,000 = x, \quad \textit{2c: } \log_x 64 = 3, \quad \textit{2d: } \log_{16} x = 1/4, \quad \textit{2e: } \log_x 9 = 1/2.$$

Using inverse properties of logs and exponents

Exercise 3:

Simplify each expression.

$$\textit{3a: } \log_5 5^{2x+7}, \quad \textit{3b: } \log 10^{5x-2}, \quad \textit{3c: } \ln e^{5-x},$$
$$\textit{3d: } \log_3 9^x, \quad \textit{3e: } \log_2 (1/2)^{3x+7}$$

Using properties of logs

Exercise 4:

Expand each expression into multiple logarithms having single character arguments (if possible).

$$\textit{4a: } \log_4(xy), \quad \textit{4b: } \log(3x^2), \quad \textit{4c: } \log_3(6x+2),$$
$$\textit{4d: } \ln\left(\frac{1}{3x}\right), \quad \textit{4e: } \ln\left(\frac{5x^4}{2\sqrt{2}}\right), \quad \textit{4f: } \log_4\left(\frac{x^3+8x^2+15x}{x+3}\right).$$

Exercise 5:

Condense each expression into a single logarithm.

$$\textit{5a: } \log_5 x + 2\log_5 y, \quad \textit{5b: } \log_2 4x - 3\log_2(2y), \quad \textit{5c: } (1/2)\ln(x^3) - \ln x.$$

Exercise 6:

Find a numerical value for each expression.

$$\textit{6a: } (1/3)\log_2 64, \quad \textit{6b: } \log 25 + \log 4, \quad \textit{6c: } \log_6 9 + \log_6 12 - \log_6 3.$$

Using logs to solve exponential equations

Exercise 7:

Solve each expression for x.

$$\textit{7a: } 5^{2x} + 15 = 28, \quad \textit{7b: } 3(6^{3x+5} - 6) = 15, \quad \textit{7c: } 7^x = 6^{2x+3}, \quad \textit{7d: } 3^x = 4^x \textit{ trick question.}$$

Using exponents to solve logarithmic equations

Exercise 8:

Solve each expression for x.

$$\textit{8a: } \log_2(3x+2) = 5, \quad \textit{8b: } 14 + 3\log_3 x = 20,$$
$$\textit{8c: } \log_4(2x+5) - \log_4(x-1) = 2, \quad \textit{8d: } 2\ln(x-3) - \ln(21-2x) = 0.$$

Using the change of base formula

Exercise 9:

Rewrite each expression as either a single natural log or exponent.

$$\textit{9a: } \log_5 x, \quad \textit{9b: } 3\log_7 x, \quad \textit{9c: } 4^x, \quad \textit{9d: } 6^{3x+2}.$$

Challenge problem

Exercise 10:

$$\textit{Solve for } x\textit{:} \quad \frac{\ln 8}{\ln x} = \frac{\ln x^9}{\ln 8}$$