Sec 2.4 Sigma notation HW + notes

Types of problems:

I. Straight summing of terms.
Show you understand how to read Σ notation.

Ex $\displaystyle\sum_{i=1}^{6} 7i = 7(1)+7(2)+7(3)+\cdots+7(6)$

$= 7(1+2+3+4+5+6)$
$= 147$

II Know the formula that gives the answer without summing to the first "n" natural numbers (Gauss' formula)

Ex Formula: $\displaystyle\sum_{i=1}^{n} i = 1+2+\cdots+n$

$$= \frac{n(n+1)}{2}$$

Ex $\displaystyle\sum_{i=1}^{9} i = \frac{9(9+1)}{2} = \frac{90}{2} = 45$

III Know some simple properties of Σ:

$$\sum_{i=1}^{n}(a_i+b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum c a_i = c \sum a_i$$

Ex $\sum_{i=1}^{8}\left(5i + \frac{i}{2}\right) = 5\sum_{i=1}^{8} i + \frac{1}{2}\sum_{i=1}^{8} i$

Notice that this approach often leads to a form you can apply Gauss' or other formula to:

$$5\sum_{i=1}^{8} i + \frac{1}{2}\sum_{i=1}^{8} i = ?$$

$$5\left(\frac{8(8+1)}{\cancel{2}}\right) + \frac{1}{2}\left(\frac{8(8+1)}{2}\right) = 5(36) + \frac{1}{2}(36)$$

$$= \underline{198}$$

III The sum of a constant from $i=1$ to n: Take this on faith.

$$\sum_{i=1}^{n} c = nc$$

Ex $\sum_{i=1}^{38} \frac{1}{4} = 38\left(\frac{1}{4}\right) = \frac{38}{4} = 9\frac{1}{2}$

The sum of a constant from $i=m$ to n:

$$\sum_{i=m}^{n} c = \underbrace{(n-m+1)}_{\text{because there's one more addend than } n-m} c$$

$\underline{\underline{Ex}}$ $\sum_{n=5}^{20} 2 = (20-5+1)(2) = \underline{\underline{32}}$

IV. That last type is a version of finding any sum from $i = m$, where $m > 1$. It's this:

$$\sum_{i=m}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{m-1} a_i$$

This is easier to understand by example, which is how we'll approach it + remember it:

$$\underline{\underline{Ex}} \quad \sum_{i=6}^{30} i = \underbrace{\sum_{i=1}^{30} i}_{\text{the first } 30\ i} - \underbrace{\sum_{i=1}^{5} i}_{\text{the first } 5\ i}$$

If we have it in this form, where i starts at 1 for both sums, we can apply Gauss' formula:

$$\sum_{i=1}^{30} i = \frac{30(31)}{2} = 15(31), \qquad \sum_{i=1}^{5} i = \frac{5(6)}{2} = 15$$

So $\sum_{i=6}^{30} i = 15(31) - 15 = 15(31-1)$

$= 15(30) = \underline{\underline{450}}$

V Changing the index so you have equivalent sums:

Ex $\sum_{i=1}^{n} a_i \longrightarrow \sum_{i=0}^{n-1} a_{i+1}$

To begin counting at zero, you'd go only as high as $n-1$ to produce the same sum. You would index the addends a_i accordingly so you're adding the same terms: As the counter goes down, the index goes up.

Ex $\sum_{i=1}^{4} 5i = \sum_{i=3}^{6} 5(i-2)$

Because I reindexed the "counter" by $+2$, I had to reindex the addend by -2.

VI Finally, the two formulas you should eventually memorize:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$