HW Quiz

HW - Curve sketching, behavior of fcn. at vertical asymptotes and at infinity, and min / max information

$$f(x) = \frac{1}{4-x^2}$$

Dom: $x \neq \pm 2$ $\quad (-\infty,-2) \cup (-2,2) \cup (2,\infty)$

V.A $\quad x = 2, \; x = -2$

H.A $\quad \lim_{\substack{x \to \infty \\ -\infty}} f(x) = 0$ since deg numerator < deg denominator

$f(0) = \frac{1}{4}$, y-int: $(0, \frac{1}{4})$

$f(x) \neq 0$, so no x-int.

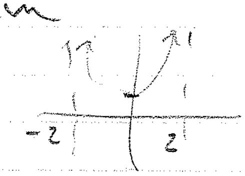
$f'(x) = \dfrac{2x}{(4-x^2)^2}$, $\quad f''(x) = \dfrac{8+6x^2}{(4-x^2)^3}$ (see next page for computation)

$f'(x) = 0$ at $x = 0$
This is the only crit. #.
Where $f'(x)$ DNE happens to be same values of x where $f(x)$ is not defined, i.e. $x = \pm 2$ is not in Dom f so they are not crit values

$f''(0) = \dfrac{8}{4^3} > 0$

the fcn. is concave up at $x=0$, hence $f(0)$ is local minimum

$f(0) = 1/4$



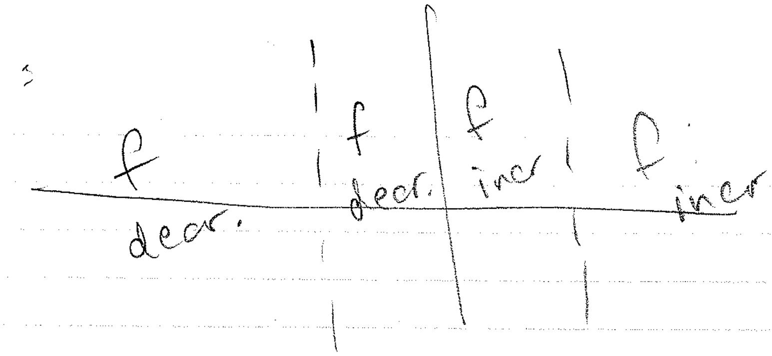
Graph so far (rough sketch)

FDT: For f increasing, decreasing, test values on $(-\infty,-2)$, $(-2,0)$, $(0,2)$, $(2,\infty)$

$f'(-3) = -/+ > 0$, $f \downarrow$ on $(-\infty,-2)$
$f'(-1) = -/+ > 0$, $f \downarrow$ on $(-2,0)$
$f'(1) = +/+ > 0$, $f \uparrow$ on $(0,2)$
$f'(3) = +/+ > 0$, $f \uparrow$ on $(2,\infty)$

Even tho' $x = \pm 2$ are not crit. numbers, we still inspect on either side because they are asymptotes.

So far:



We have much more info than this, so you could already sketch the function.

But, on the test you will need to show you can inspect for concavity by means of the SDT.

$$f'(x) = \frac{2x}{(4-x^2)^2}, \quad f''(x) = \frac{2(4-x^2)^2 - 2(4-x^2)'(-2x)(2x)}{(4-x^2)^4}$$

Simplifying by factoring out $(4-x^2)'$

$$f''(x) = \frac{2(4-x^2)' + 8x^2(1)}{(4-x^2)^3} = \boxed{\frac{8+6x^2}{(4-x^2)^3}}$$

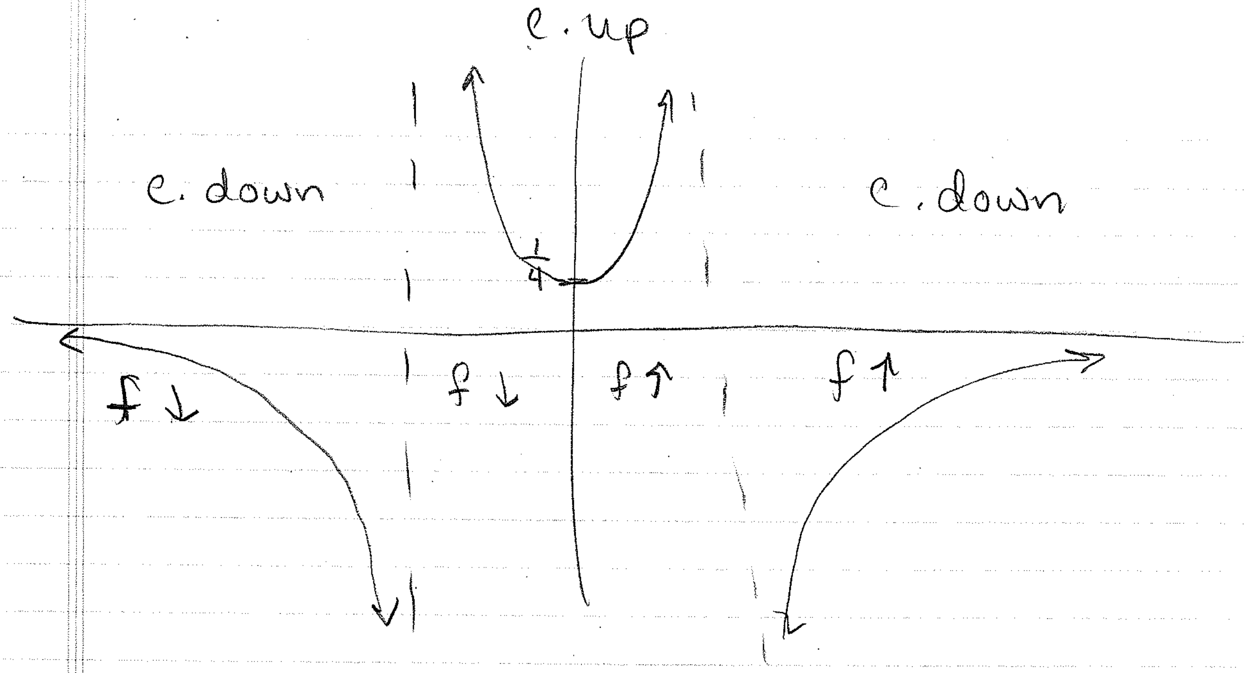
What is concavity at $x=0$ (the only crit. number)?

$$f''(0) = \frac{8+0}{(4-0)^3} > 0,$$ hence $f(x)$ is concave up at $x=0$, its local minimum, as already found, is $(0, \frac{1}{4})$

Test concavity left of $x=-2$ and right of $x=2$ (we already know that it's concave up on $(-2,2)$ since $x=0$ has a local min rather than an inflection point)

$$f''(-3) = \frac{8+6(-3)^2}{(4-(-3)^2)^3} = \frac{+}{-} < 0,$$ hence concave down on $(-\infty, -2)$

$$f''(3) = \frac{8+6(3)^2}{(4-3^2)^3} = \frac{+}{-} < 0,$$ also concave down on $(2, \infty)$



Note the HA of $y=0$. How can we be sure the graph doesn't cross the HA?

The fcn. is decreasing and always negative on $(-\infty,-2)$ and on $(2,\infty)$ $\left[f(x)=\frac{1}{4-x^2}<0 \text{ when } x^2>4,\right.$ that is, $\left.x<-2 \text{ or } x>2\right]$

You could always plot $f(-3)$ and $f(3)$ to get an insight on sign of $f(x)$ there.

<u>Summary</u> For test, you will need to give all the information on domain, intercepts, asymptotes, intervals of ↑, ↓, concave up, concave down, and POI.

Since $f''(x)\neq 0$, there is no POI.

Final note: If $f''(x)$ did = zero at some value of x, you cannot assume POI. SDT is needed.