

Sle 8.8

#1) Requires sum-to-product formula, so you may skip.

#2) Show  $\sin(\alpha + 2\pi) = \sin \alpha$

using sum formula:

$$\begin{aligned}\sin(\alpha + 2\pi) &= \sin \alpha \cos(2\pi) + \sin(2\pi) \cos \alpha \\ &= \sin \alpha \cdot 1 + 0 \cdot \cos \alpha \\ &= \sin \alpha \quad \checkmark\end{aligned}$$

#3) Verify  $(\sin x + \cos x)^2 = 1 + \sin(2x)$

On left:  $\sin^2 x + 2 \sin x \cos x + \cos^2 x$

$$\begin{aligned}&= 1 + 2 \sin x \cos x \\ &= 1 + \underline{\sin 2x} \quad (\text{Double angle formula})\end{aligned}$$

#4) Evaluate using sum + diff formulas:

a)  $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right)$

from double angle formula

$$\boxed{\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}}$$

b)  $\sin\left(\frac{\pi}{5}\right) \cos\left(\frac{3\pi}{10}\right) + \cos\left(\frac{\pi}{5}\right) \sin\left(\frac{3\pi}{10}\right)$

from sum formula

$$= \sin\left(\frac{\pi}{5} + \frac{3\pi}{10}\right)$$

$$= \sin\left(\frac{5\pi}{10}\right)$$

$$= \sin(\pi/2) = \boxed{1} \quad //$$

$$c) [\tan(x + \pi)] [\tan(x - \pi)] + 13 = ?$$

Use Sum + Diff formula for tan:

Substituted

$$[\tan(\alpha + \beta)][\tan(\alpha - \beta)] \quad \text{part}$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \cdot \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Substitute  $x$  &  $\pi$  for  $\alpha$  &  $(x + \beta)$ :

~~Substitute  $x$  &  $\pi$  for  $\alpha$  &  $(x + \beta)$ :~~

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} \cdot \frac{\tan x - \tan \pi}{1 + \tan x \tan \pi}$$

$$= \frac{\tan^2 x - \tan^2 \pi}{1 - \tan^2 x \tan^2 \pi}$$

$$= \frac{\tan^2 x - 0}{1 - (\tan^2 x) 0} = \frac{\tan^2 x}{1} = \tan^2 x$$

Given that  $+1$  is added to this:

$$\tan^2 x + 1 = \boxed{\sec^2 x} \quad \text{from identity II}$$

$$\#4d) \frac{\tan\left(\frac{\pi}{5}\right) - \tan\left(\frac{\pi}{30}\right)}{1 + \tan\left(\frac{\pi}{5}\right)\tan\left(\frac{\pi}{30}\right)} = \tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right)$$

↑

from diff formula  
for tangent

$$\tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right) = \tan\left(\frac{6\pi - \pi}{30}\right)$$

$$= \tan\left(\frac{5\pi}{30}\right) = \tan\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{\sqrt{3}}} \quad //$$

$$\#6 \quad \sin(3x) = ? \quad 3 \sin x - 4 \sin^3 x$$

$$\text{Note: } \sin(3x) = \sin[(2x) + (x)]$$

Let's try using ~~the~~ formulas:

$$\textcircled{1} \sin(2x) = 2\sin x \cos x, \textcircled{3} \sin(x+y) =$$

$$\textcircled{2} \cos(2x) = \cos^2 x - \sin^2 x, \quad \sin x \cos y + \sin y \cos x$$

Then:

$$\sin(3x) = \sin[(2x) + x] \quad \text{using}$$

$$= \sin(2x) \cos(x) + \sin(x) \cos(2x)$$

$$= 2\sin x \cos x \cos x + (\sin x)(\cos^2 x - \sin^2 x)$$

$$= 2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x$$

$$= 2\sin x (1 - \sin^2 x) + \sin x (1 - \sin^2 x) - \sin^3 x$$

$$= 2\sin x - 2\sin^3 x + \sin x - \sin^3 x$$

$$= 3\sin x - 4\sin^3 x // \quad \text{Wow! I did it!}$$

And so can you ☺

$$48) \text{a) } \cos(105^\circ) = \cos(60^\circ + 45^\circ)$$

$$\begin{aligned} &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2}, \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2} - \sqrt{6}}{4} // \end{aligned}$$

$$\text{b) } \sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{8\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right) \cos\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4} //$$

$$\text{c) } \sin(195^\circ) = \sin(150^\circ + 45^\circ)$$

$$= \sin 150^\circ \cos 45^\circ + \sin 45^\circ \cos 150^\circ$$

$$= \sin 30^\circ \cos 45^\circ + (\sin 45^\circ)(-\cos 30^\circ)$$

$$= \frac{1}{2}, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}, \left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$