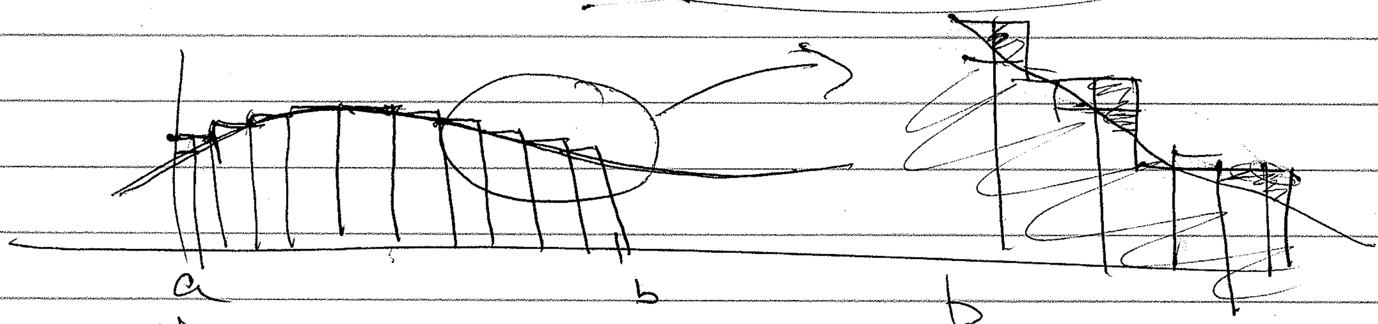
POST

Sec. 35 is 3 sections

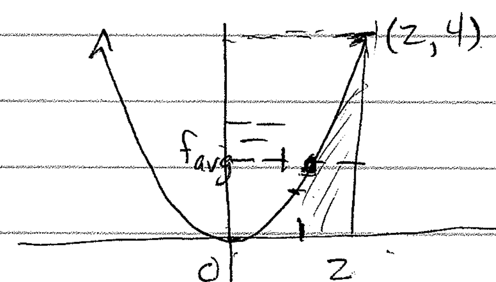
I Riemann sum on [a,b] $= \lim_{N \to \infty} \sum_{i=1}^{N} f(x_i) \Delta x = \int f(x)\,dx$



$$\lim_{N\to\infty} \sum_{i=1}^{N} \begin{pmatrix}\text{height}\\ \text{of}\\ \text{rect } i\end{pmatrix} (\text{width}) = \int_a f(x)\,dx$$

$f(x_i)$ stays the same Δx

Ex 35.1 Avg value of $f(x) = x^2$ on $[0,2]$



$f(x) = x^2$

$f(2) = 4$

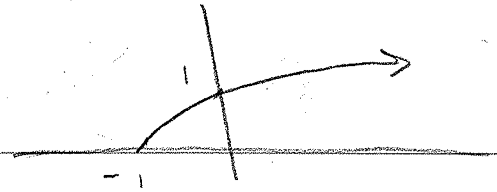
The avg value of the fcn on $[0,2]$ is $4/3$

$$\text{Area} = \int_0^2 x^2\,dx = \left.\frac{x^3}{3}\right|_0^2 = \frac{8}{3}$$

$$\text{Avg value} = \frac{1}{b-a}(\text{area}) = \frac{1}{2-0}\cdot\frac{8}{3} = \frac{8}{6} = \frac{4}{3}$$

Sec 35 #1 continued

#1d) $f(x) = \sqrt{x+1}$ on $[1,2]$



$$\underset{\text{on } [1,2]}{f_{avg}} = \frac{1}{2-1}\int_1^2 \underbrace{\sqrt{x+1}}_{u^{1/2}}\;\underbrace{dx}_{du} = \frac{2(x+1)^{3/2}}{3}\Big|_1^2$$

$$= \frac{2}{3}\left(3^{3/2} - 1^{3/2}\right) = \frac{2}{3}\left(3^{3/2} - 1\right)$$ This is

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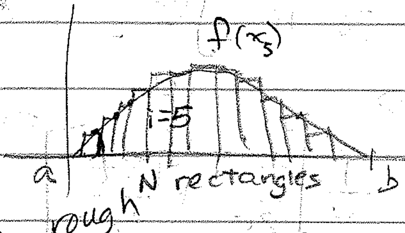
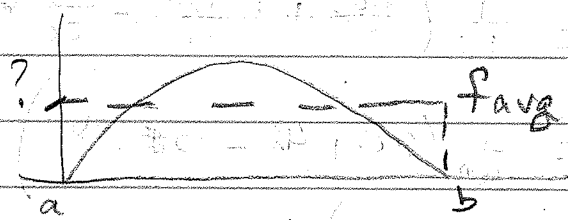
Sec 35 – Several formulas + applications

II. Average value of a function on $[a,b]$



height of each rect "i" = $f(x_i)$
width of each rect $= \Delta x$ is constant

The rough sum of fcn. values is $\sum_{i=1}^{N} f(x_i)$, where $f(x_i)$ is the ith the fcn. value at the top of ~~each~~ rectangle.



f_{avg} occurs at $\dfrac{\sum f(x_i)}{N}$ (add up fcn values, divide by N)

Notice that $\dfrac{\sum f(x_i)\cdot \Delta x}{N\cdot \Delta x} = \dfrac{\sum f(x_i)}{N}$

and that $b-a = N\Delta x$, the sum of lengths of rect bases
Hence, the average value of the fcn on $[a,b]$ is roughly

$$\frac{1}{N\Delta x}\sum_{i=1}^{N} f(x_i)\Delta x = \frac{1}{b-a}\sum_{i=1}^{N} f(x_i)\,\Delta x$$ Rough est of avg value

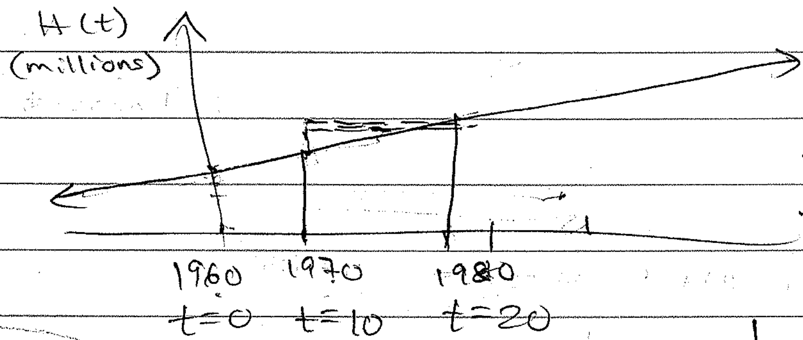
The exact average value of f on $[a,b]$ is ~~found~~ by taking the limit of these rect. areas (i.e., the integral $\int f(x)dx$) and multiplying by $\frac{1}{b-a}$

2.
$$\lim_{N\to\infty} \frac{1}{b-a}\sum_{i=1}^{N} f(x_i)\Delta x =$$

$$\text{Avg value of } f(x) \text{ on } [a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx$$ ☆

Hw Sec 3.5

#3. $H(t) = \frac{1}{10}t + \frac{12}{5}$



$$\frac{1}{20-10}\int_{10}^{20}\left(\frac{1}{10}t + \frac{12}{5}\right)dt$$

$$\frac{1}{10}\left[\frac{t^2}{20} + \frac{12t}{5}\right]_{10}^{20}$$

$$= \frac{1}{10}\left(\frac{20^2}{20} + \frac{12(20)}{5} - \frac{10^2}{20} - \frac{12(10)}{5}\right)$$

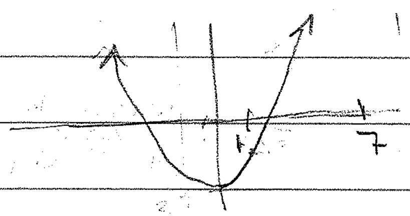
$$= \frac{1}{10}\left(20 + 48 - 5 - 24\right)$$

$$= \frac{1}{10}(39) = \boxed{3.9 \text{ million hamburgers}}$$

Avg value of $f(x)$ on $[a,b]$ $= \frac{1}{b-a}\int_a^b f(x)\,dx$

#2

#1b) $f(x) = x^2 - 3$ Find f_{avg} on $[1,7]$



Notice the interval straddles a root
($x^2 - 3 = 0 \rightarrow x = \pm\sqrt{3}$)

Does this require we split up the integral?
No, since we are not considering area

Hence, $f_{avg} = \frac{1}{7-1}\int_1^7 (x^2-3)\,dx = \frac{1}{6}\left[\frac{x^3}{3} - 3x\right]_1^7$ notice the use of brackets

$$= \frac{1}{6}\left[\left(\frac{7^3}{3} - 3\cdot 7\right) - \underbrace{\left(\frac{1}{3} - 3\cdot 1\right)}_{-2\,2/3}\right] = \frac{1}{6}\left(\frac{7^3}{3} - 21 + \frac{8}{3}\right)$$

$$= \frac{1}{6}\left(\frac{343 - 63 + 8}{3}\right) = \frac{96}{6} = \boxed{16}$$

Sec 35 continued (topic III)

Present + Accumulated Value - Previously, we considered an investment of a principal at some annual rate of interest, compounded continuously, and we sought its value at some future time.

$$FV = Pe^{rt}$$

This scenario assumes a one time input of money, with growth on interest and principal alone. More realistically, money is flowed into an investment throughout the years of investment.

Without continuous flow, the above formula solved for P would tell us the "present value" of an investment that at some future time t is worth FV.

$$P = FVe^{-rt}$$

That is, to attain a given FV at some rate + time of investment, you'd need $P = FVe^{-rt}$ principal. This "P" we now call "present value"

$$PV = FVe^{-rt}$$

If money is flowing continuously at some $f(t)$, the analogous formulas are these:

$$PV = \int_0^T f(t)\, e^{-rt}\, dt \quad , \qquad \boxed{FV = \int_0^T f(t)\, e^{r(T-t)}\, dt}$$

given amt

How it generates income

income flow - often a constant stream

Calculate this when you need to compare your business investment to an offer to buy, i.e., fair market price

Generally the scenario for bank investment (deposits)

PV is more common for deciding to sell or keep a business

FV is the choice for looking at value of an investment