Sec 32 Integration by Parts

Remember $\frac{d}{dx}\left[f \cdot g\right] = f'g + g'f$

Consider 2 functions, $u(x)$, $v(x)$

where $\frac{d}{dx}(u \cdot v) = v\frac{du}{dx} + u\frac{dv}{dx}$

Solve for $u\,dv/dx$: $u\frac{dv}{dx} = \frac{d(uv)}{dx} - v\frac{du}{dx}$

Take integral of both sides with respect to x: $\int u\frac{dv}{dx} \cdot dx = \int \frac{d\,uv}{\cancel{dx}} \cdot \cancel{dx} - \int v\frac{du}{\cancel{dx}}\cancel{dx}$

$\int d(uv)$

$\int u\frac{dv}{\cancel{dx}}\cancel{dx} = uv - \int v\,du$

KNOW !!!

$$\boxed{\int u \cdot dv = uv - \int v\,du}$$

Try to make your given ~~integrals~~ integrand of some form of product look like this expression

Ex 1. $\int x e^x \, dx \longleftrightarrow \int u \, dv$

Name u, dv, say

Attempt 1 let $\boxed{\begin{array}{l} u = x \\ du = dx \end{array}}$ + $\int dv = \int e^x dx$

from $\boxed{\begin{array}{l} v = e^x \\ dv = e^x dx \end{array}}$

Ingredients to put into the processor $\int u \, dv$

$$\int x e^x dx = x e^x - \int e^x dx$$

Answer $\boxed{= x e^x - e^x + C}$ ✓

Attempt 2 let $\boxed{u = e^x}$, $\int dv = \int x \, dx$

not needed $\boxed{du = e^x dx,}$ $\boxed{v = \frac{x^2}{2}}$

$$\int x e^x dx = \frac{e^x \cdot x^2}{2} - \int \frac{x^2}{2} \cdot e^x dx$$

Wrong approach

Even worse - not integrable

Ex 2 $\quad \int \frac{1-x}{3e^{x}}\,dx = \frac{1}{3}\int (1-x)e^{-x}\,dx$

$u = 1-x \qquad\qquad dv = e^{-x}dx$

$\frac{du}{dx} = -x \qquad\qquad \int dv = \int e^{-x}dx = -e^{-x}$

$\boxed{\text{let } w = -x}$

$du = -x\,dx \qquad\qquad e^{w}\,dw\ ?$

$\frac{dw}{dx} = -1$

$dw = -1\,dx$

So, $-\int e^{-x} \underbrace{dx}_{dw} = -\int e^{w}\,dw$

we choose $\quad u = 1-x$

we diff $\quad du = -dx$

we choose $\quad dv = e^{-x}dx$

we int $\quad \boxed{v = -e^{-x}}$

$\frac{1}{3}\int u\,dv = uv - \int v\,du$

$\frac{1}{3}\int u\,dv = \cancel{(1-x)e^{-x} \int e^{-x}dx}$

$= \cancel{(1-x}$

$= \frac{1}{3}\left((1-x)(-e^{-x}) - \int -e^{-x}(-dx)\right)$

$= \frac{1}{3}\left(-(1-x)e^{-x} + \int -e^{-x}dx\right)$

$= \quad \frac{1}{3}e^{-x}$

Review — basic u-sub $\int e^u du$

$\int e^{2x} dx \overset{\text{form}}{=} \int e^u du = e^u + C$

Ex 1

let $\boxed{u = 2x}$, $du = 2dx$

or

$\boxed{dx = \frac{du}{2}}$

$\int \frac{e^u du}{2}$

$= \frac{1}{2} \int e^u du$

$= \frac{1}{2} e^u + C = \frac{1}{2} e^{2x} + C$

Ex 2 $\int e^{-x} dx$ let $u = -x$

$du = -1 dx$

so $dx = -du$

$\int e^u (-du)$

$= -\int e^u du$

$= -e^u + C = -e^{-x} + C$

$$\boxed{\underline{\text{Memorize}} \quad \int e^{ax} dx = \boxed{\frac{1}{a} e^{ax} + C}}$$