Sec 32 Integration by Parts – HW

Form: $\int u\,dv = uv - \int v\,du$

1a) $\int (x+1)e^x\,dx$

Let $u = x+1$, $\quad dv = e^x\,dx$

Then $du = 1\,dx$, $\quad v = \int dv = e^x$

Substitute into $\int u\,dv = uv - \int v\,du$

$$\int (x+1)e^x\,dx = (x+1)e^x - \int e^x\,dx$$
$$= (x+1)e^x - e^x + C$$
$$= xe^x + e^x - e^x + C = \boxed{xe^x + C}$$

b) $\int 2xe^x\,dx = 2\int xe^x\,dx$

Again, let $u = x$ and $dv = e^x\,dx$
(i.e., choose the polynomial term to be u & exp to be dv)

Then $du = dx$ and $v = \int e^x\,dx = e^x$

$$2\int xe^x\,dx = 2\left[xe^x - \int e^x\,dx\right]$$
$$= 2xe^x - 2e^x + C = \boxed{2e^x(x-1) + C}$$

1c) $\int \underbrace{(x-3)}_{u} \underbrace{e^{3x}\,dx}_{dv}$ $\quad du = dx$

$v = \frac{e^{3x}}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$= (x-3)\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\,dx$$

$$= \frac{xe^{3x}}{3} - e^{3x} - \frac{1}{3}\cdot\frac{e^{3x}}{3} + C$$

$$= \frac{xe^{3x}}{3} - e^{3x} - \frac{e^{3x}}{9} + C$$

$$= \boxed{\frac{xe^{3x}}{3} - \frac{10e^{3x}}{9} + C}$$

d) $\int x e^{-x/5}\,dx$ $\quad$ Try $u = x$ $\quad dv = e^{-x/5}$

$du = dx \quad v = -5e^{-x/5}$

$$\int xe^{-x/5}\,dx = -5xe^{-x/5} - \int -5e^{-x/5}\,dx$$

$$= -5xe^{-x/5} + 5\int e^{-x/5}\,dx$$

w, dw $-\frac{1}{5}dx$

$$= -5xe^{-x/5} + 5(-5)\int\left(-\frac{1}{5}\right)e^{-x/5}\,dx$$

$$= -5xe^{-x/5} - 25\int e^{w}\,dw$$

$$= -5xe^{-x/5} - 25e^{w} + C$$

$$= \boxed{-5xe^{-x/5} - 25e^{-x/5} + C}$$

1e) First try this by parts;

$$\int \ln x \, dx \qquad \text{let } u = \ln x \qquad du = \frac{dx}{x}$$
$$dv = dx \qquad v = x$$

$$\int \ln x \, dx = x \ln x - \int x \frac{du}{x} = \boxed{x \ln x - x}$$

It's noteworthy that the $\ln x$ term is the u term.

Now, 1e: $\int x^2 \ln x \, dx$ let $u = \ln x$ $\quad du = \frac{dx}{x}$

$dv = x^2 dx \quad v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \frac{dx}{x} = \frac{x^3}{3} \ln x - \frac{1}{3}\int x^2 dx$$

$$= \boxed{\frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C} \text{ or } \boxed{\frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + C}$$

g) $$\int \frac{1-x}{3e^x} dx = \frac{1}{3}\int \underbrace{\underbrace{e^{-x}}\,\underbrace{(1-x)}_{u}\,\underbrace{dx}}_{dv}$$

$u = 1 - x$, $du = -dx$

$dv = e^{-x} dx$, $v = -e^{-x}$

$$\frac{1}{3}\left[(1-x)(-e^{-x}) - \int -e^{-x}(-dx)\right]$$

$$= \frac{1}{3}\left[-e^{-x} + xe^{-x} - \int e^{-x} dx\right]$$

$$= \frac{1}{3}\left[-e^{-x} + xe^{-x} + e^{-x} dx\right] + C = \boxed{\frac{1}{3} xe^{-x} + C}$$

1h) $\int \ln(2x)\,dx$ $\quad u = \ln(2x),\quad du = \frac{1}{2x}\cdot 2\,dx = \frac{dx}{x}$

$dv = dx,\quad v = x$

$$\int \ln(2x)dx = x\ln(2x) - \int \cancel{x}\frac{dx}{\cancel{x}} = x\ln(2x) - x + C$$

1i) $\int \frac{\ln x}{x^2}\,dx$ $\quad u = \ln x,\quad du = \frac{dx}{x}$

$dv = \frac{dx}{x^2},\quad v = -\frac{1}{x}$

$$\int \frac{\ln x}{x^2}dx = -\frac{\ln x}{x} - \int -\frac{1}{x}\frac{dx}{x}$$

$$= -\frac{\ln x}{x} + \int x^{-2}dx = -\frac{\ln x}{x} - x^{-1} + C$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C = -\frac{1}{x}(\ln x + 1) + C$$

1k) $\int (2x+9)e^x\,dx$ $\quad u = 2x+9,\quad dv = e^x dx$

$du = 2\,dx,\quad v = e^x$

$$\int (2x+9)e^x dx = (2x+9)e^x - \int e^x \cdot 2dx = (2x+9)e^x - 2e^x + C$$

$$= 2xe^x + 9e^x - 2e^x + C = \boxed{2xe^x + 7e^x + C}$$