Sec 31

1a) $\int (3x+1)^5\,dx$ $\qquad$ $u = 3x+1$, $du = 3\,dx$ or $dx = \frac{du}{3}$ $\to$ $\int \frac{u^5\,du}{3}$

$= \frac{1}{3}\frac{u^6}{6} + C$

$\frac{1}{18}(3x+1)^6 + C$ or $\boxed{\frac{(3x+1)^6}{18} + C}$

Alternately: Since $du = 3\,dx$, you need a 3 inside the integral, so mult by 3 inside, divide by 3 outside:

$\frac{1}{3}\int \underbrace{3(3x+1)^5\,dx}_{du} = \frac{1}{3}\int u^5\,du = \frac{1}{3}\cdot\frac{u^6}{6} + C$ etc.

"1 (adjustment doesn't affect value)

b) $\int (-t+1)^3\,dt = -\int \underbrace{-(-t+1)^3\,dt}_{du}$ $\qquad$ $u = -t+1$, $du = -dt$

So $-\int u^3\,du = -\frac{u^4}{4} + C = \boxed{-\frac{(-t+1)^4}{4} + C}$

c) $\int \sqrt{4x-1}\,dx = \int (4x-1)^{1/2}\,dx = \frac{1}{4}\int (\underbrace{4x-1}_{u})^{1/2}\cdot \underbrace{4\,dx}_{du}$

$= \frac{1}{4}\int u^{1/2}\,du = \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C = \frac{1}{6}u^{3/2} + C$

$= \boxed{\frac{1}{6}(4x-1)^{3/2} + C}$

1 e) $\int \underbrace{x(x^2+1)^3\,dx}_{du/2}$ let $u = x^2+1$, $du = 2x\,dx$

then $x\,dx = \frac{du}{2}$

$$\int u^3 \frac{du}{2} = \frac{1}{2}\int u^3\,du = \frac{1}{2}\frac{u^4}{4} + C$$

$$= \frac{1}{2}\cdot\frac{1}{4}(x^2+1)^4 + C$$

$$= \boxed{\frac{(x^2+1)^4}{8} + C}$$

f) $$\int 4e^{2z}\,dz = 2\int \underbrace{2e^{\overset{u}{2z}}\,dz}_{du} = 2\int e^u\,du$$

$$= 2e^u + C = \boxed{2e^{2z} + C}$$

g) $\int \frac{2x^4}{x^5+1}\,dx$ let ~~$u = x^5+1$~~ $x^5+1 = u$

$5x^4\,dx = du$

$$= 2\int \frac{x^4}{x^5+1}\,dx = 2\cdot\frac{1}{5}\int \frac{5x^4}{x^5+1}\,dx$$

$$= \frac{2}{5}\int \frac{du}{u} = \frac{2}{5}\ln|u| + C$$

$$= \boxed{\frac{2}{5}\ln|x^5+1| + C}$$

1 i) $\int x(x-2)^5\,dx$ Tricky.

If $u = x-2$
then $x = u+2$ } $\to$ Two u-subs at once!
and $dx = du$

gives $\int (u+2)\,u^5\,du$

$= \int (u^6 + 2u^5)\,du$ (Did not see this coming :))

$= \frac{u^7}{7} + \frac{2u^6}{6} + C = \boxed{\frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C}$

k) $\int \frac{\ln x}{x}\,dx$ let $u = \ln x$

$du = \frac{1}{x}\,dx$

giving $\int u\,du = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C}$

A very good focus problem!