Sec 30 - The Antiderivate (Indefinite Integral)

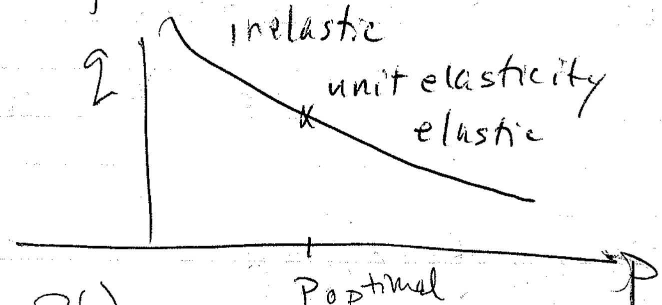
First, these items from before

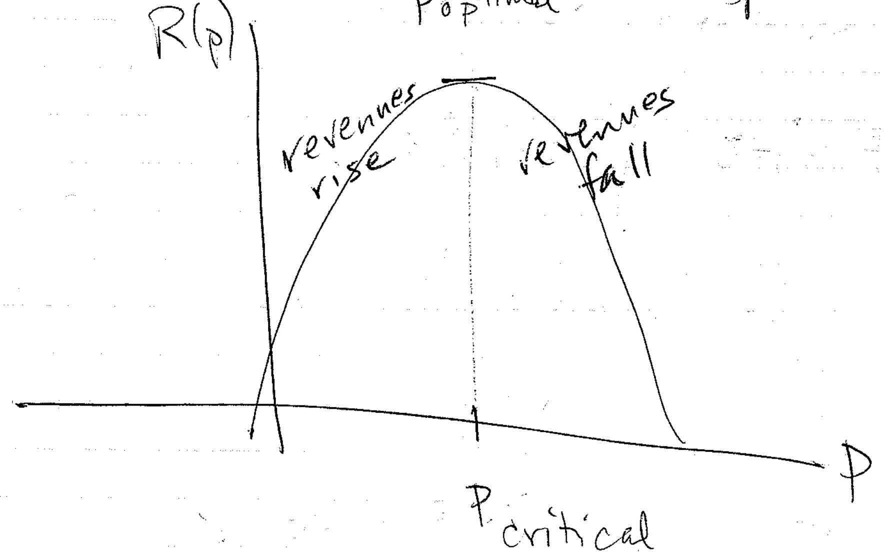
- Curve sketching - an error on derivatives will mess up entire problem. Esp. quotient rule troubles. (Mostly algebra, but also some confusion seen in formula application $\frac{u'v - v'u}{v^2} = \frac{d(u/v)}{dx}$)

- Elasticity - #3 is badly phrased - focus on

$$E(p) \begin{matrix} <1 \\ >1 \\ =1 \end{matrix} \Bigg\}$$ meanings and connection to $R'(p)$

via demand curve





$R'(p) = 0$ at same value where $E(p) = 1$

Antiderivatives

We know how to find the derivative of, say, a polynomial:

$$F(x) = x^2 - 3x + 2$$

$$F'(x) = 2x - 3 = f(x)$$

Given a polynomial, how do we go the other way? How do we find ~~the~~ an "antiderivative"? That is, given $f(x)$, how do we find an antiderivative ~~Call f(x) above F(x) for now~~. $F(x)$?

The following is the scheme of antidifferentiation

$$\cancel{F(x) = \int f(x)\,dx}$$

$$F(x) + C = \int f(x)\,dx$$

That is, a fcn. $f(x)$ ~~is the~~ has an antiderivative of the form $F(x) + C$ where C is a const.

Notice that $F(x) = x^2 - 3x + 2$

and $G(x) = x^2 - 3x - 6$

have the same derivative, namely $2x - 3$.

$F(x)$ & $G(x)$ differ from each other by a constant

$$F(x) - G(x) = (x^2 - 3x + 2) - (x^2 - 3x - 6) = 8$$

Hence $F(x) = G(x) + 8$

(you could also write $G(x) = F(x) - 8$)

Def

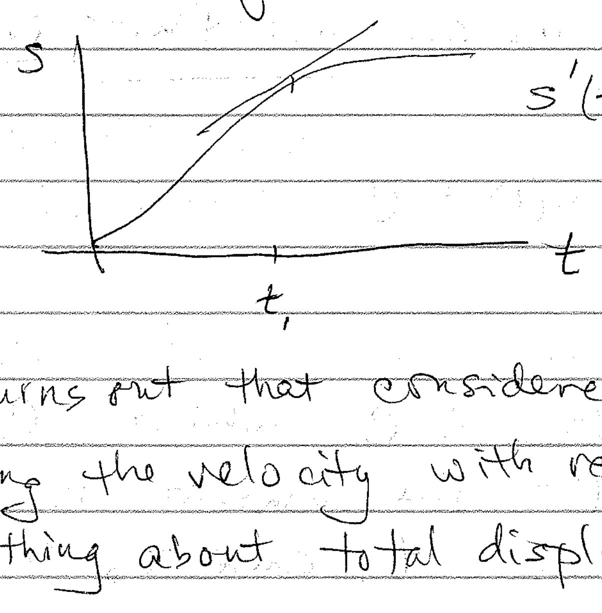
We say that fcns of the form $F(x) + C$ are ~~the~~ antiderivatives of $f(x)$, or, in "integration" symbols:

$$F(x) + C = \int f(x)\,dx$$

~~F(x)~~+C is the family of antiderivatives of $f(x)$

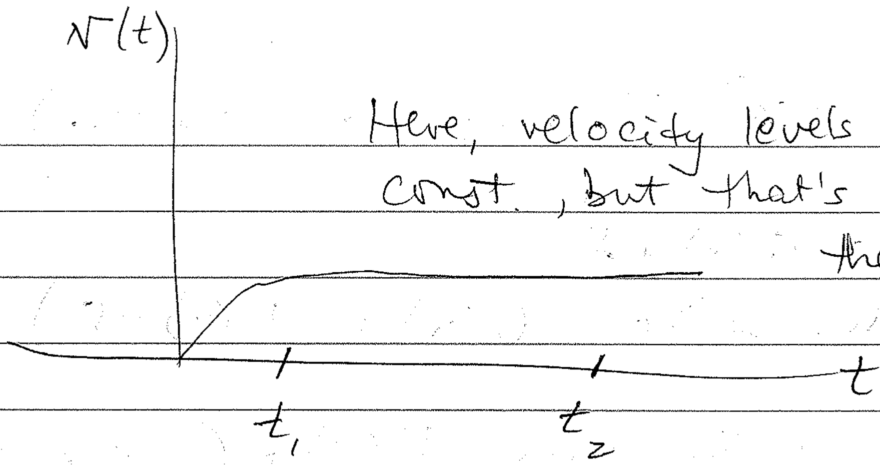
Now, this symbolism has a background. But before discussing it, consider the meaning, again, of differentiation vis-a-vis displacement fcn. $s(t)$

Given the fcn of displacement of a particle (object, body) with respect to time, ~~we~~ we found that the velocity $v(t) = s'(t)$



$s'(t_1)$ = velocity of object at time t_1; or $v(t_1)$ (slope of the tangent line)

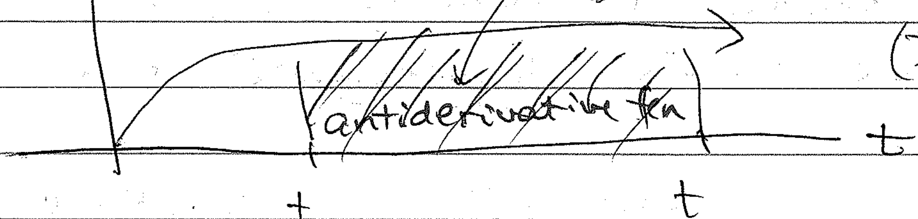
It turns out that considered the other way, ~~or the~~ graphing the velocity with respect to time, we can discover something about total displacement up to a given time.



Here, velocity levels out to some const., but that's to help us get the idea of the antiderivative

It turns out that the area under the curve from t_1 to t_2 is the total displacement $s(t_2) - s(t_1)$ of the object.

$$\int_{t_1}^{t_2} v(t)\,dt = s(t_2) - s(t_1)$$



(Don't worry about the details of $\int$ notation yet)

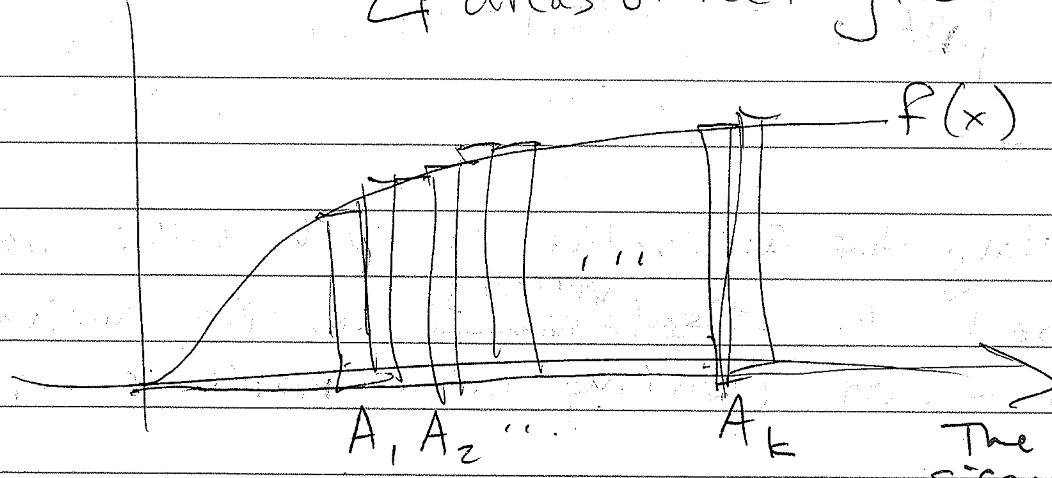
If we don't consider endpts. t_1, t_2, we can still write this quantity as an indefinite integral

$$\underset{\text{of velocity}}{\overset{\text{The anti-derivative}}{}}\int v(t)\,dt = s(t) + C$$

is the indefinite integral $\int v(t)\,dt$

Where we say the indefinite integral of velocity $v(t)$ = displacement $s(t) + C$, where "C" is a kind of catchall for the amount to be ~~determined by the values of t_1 and t_2 to be given. When we have those values, we have~~ a definite integral

Σ areas of rectangles:



$$\lim_{n\to\infty} \sum_{k=1}^{n} A_k = \int f(x)\,dx$$

The sigma becomes the squiggle

The connection between A_k and $f(x_k)$ is that $f(x_k)$ is the height of the kth rectangle and its width is the base of the rectangle, the increment h.

We don't need to worry about the detailed notation from Σ to $\int$ in this course, but the basis for $\int$ as the antiderivative symbol is crucial: it signifies the area under the curve as interpreted through limit process of the sum (Σ, $\int$) of rectangle areas.

Definite Integral – specific curve with endpts.

$$\int_{\text{begin } t_1}^{\text{stop } t_2} v(t)\,dt = s(t_2) - s(t_1)$$

$$s(t)\Big|_{t_1}^{t_2}$$

named

$$s(t)\Big|_{t_1}^{t_2} = s(t_2) - s(t_1)$$

Where, taking the derivative of each side would get us back to ~~displacement~~ velocity as the derivative of displacement (working with indefinite form again)

$$\frac{d}{dt}\int v(t)\,dt = \frac{d}{dt}\left[s(t) + C\right]$$

$$v(t) = s'(t) + 0$$

$$v(t) = s'(t)$$

Now, what's up with the $\int$ and the area?

$\int$ looks like an "ess" (letter S), and in math, "S" is often presented as the Greek sigma, Σ. You might recognize this from statistics as a "summation" symbol.

That's not a coincidence. In calculus, the sum of the areas of lots of very skinny rectangles under a curve (how skinny? their width goes to zero) is the integral of the fcn. whose curve we're considering.

Sec 30 HW #1 a-l

Review formulas

(1) **Power** $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\left(\int 1 dx = \int x^0 dx = x + C\right)$

$n \neq -1$.

ex $\int x^7 dx = \frac{x^8}{8} + C$

Check $\frac{d}{dx} \int x^7 dx = \frac{d}{dx}\left[\frac{x^8}{8} + C\right] = \frac{d}{dx}\frac{x^8}{8} + \frac{dC}{dx}$

undo $\int$ with d/dx

$x^7 \stackrel{?}{=} \frac{8x^7}{8} + 0 = x^7$ ✓

(2) Related rules · $\int f(x) \pm g(x)\, dx = \int f(x)dx \pm \int g(x) dx$

· $\int c f(x) dx = c \int f(x) dx$

But $\int \frac{f(x)}{g(x)} dx \neq \frac{\int f(x) dx}{\int g(x) dx}$

$\int f(x) g(x) dx \neq \int f(x) dx \int g(x) dx$

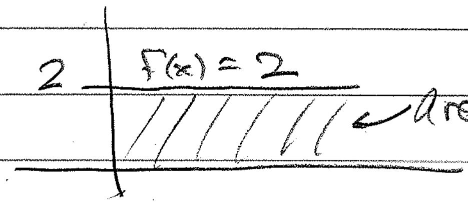
The prod + quotient integrands are dealt with – if they are integrable forms – by integration by parts (technique: " u substitution (sec 31)

(3) $\int e^x dx = e^x + C$ ← constant of integration – needed until you're given endpoints

$\int k\,dx = k\int 1\,dx = kx + C$

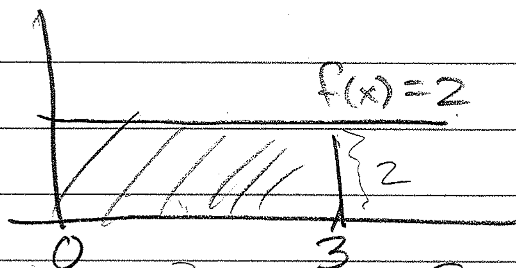
#1 HW a) $\int 2\,dx = 2x + C = F(x) + C$

Indef



Area is described by $2x + C$ where endpts are not given

Def



Area = 6

$$\int_0^3 2\,dx = 2x\Big|_0^3 = 2(3) - 2(0) = 6$$

Memorize the transition from

$$\int f(x)\,dx = F(x) + C$$ family of solns.

to

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ definite integral - finite area

antiderivative evaluated at $x = b$ minus at $x = a$

Rules continued

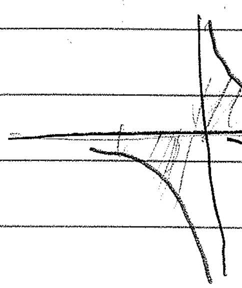
(4) $\int \frac{1}{x}\,dx \equiv \int \frac{dx}{x} = \int x^{-1}\,dx$

Power rule does not apply since $n=-1$

So, the antiderivative is:

$$\int \frac{dx}{x} = \ln|x| + C$$

Need to consider negative x, so include abs value in your answer



Dom: $x \neq 0$

So #1 i) $\int x^{1/2}\,dx + \int \frac{3}{x}\,dx - \int e^x\,dx$

$$= \frac{x^{3/2}}{3/2} + 3\ln|x| - e^x + C$$

$$= \boxed{\frac{2}{3}x^{3/2} + 3\ln|x| - e^x + C}$$

#1b) $\int (x + x^3)\,dx = \int x^1\,dx + \int x^3\,dx$

$= \frac{x^2}{2} + \frac{x^4}{4} + C$

c) $\int (12 - 3x)\,dx = \int 12\,dx - \int 3x\,dx$

$= 12\int dx - 3\int x^1\,dx$ $\qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$= 12x - \frac{3x^2}{2} + C$

d) $\int \frac{3}{\sqrt{t}}\,dt = \int \frac{3}{t^{1/2}}\,dt = 3\int t^{-1/2}\,dt$

$= \frac{3t^{1/2}}{1/2} + C = 3\cdot 2t^{1/2} + C = 6t^{1/2} + C$

e) $\int x^{2/3} + x^{-1/3}\,dx$

i) $\int \sqrt{x} + \frac{3}{x} + e^x\,dx$ rules? sum power exp

$= \int x^{1/2}\,dx + \int 3x^{-1}\,dx + \int e^x\,dx$