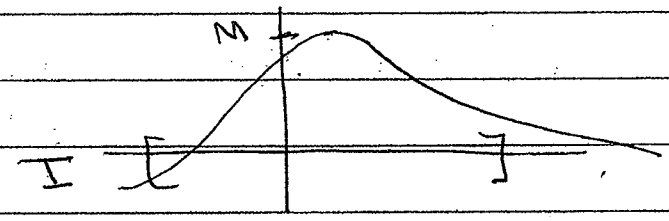
10/22

Sec 22 Absolute Max & Min

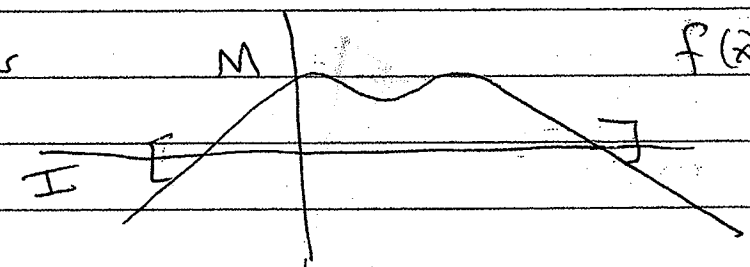
The requirement that $f(x) = M$ be an absolute max on an interval is that no other y-value of the fcn. be greater * than M.

For now let I be closed

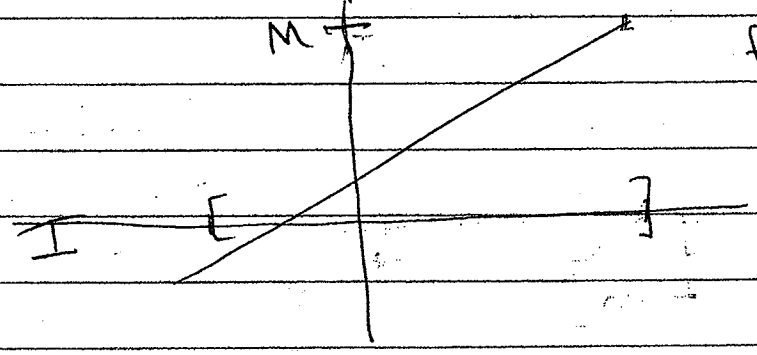


$f(x) < M$ for all $x \in I$

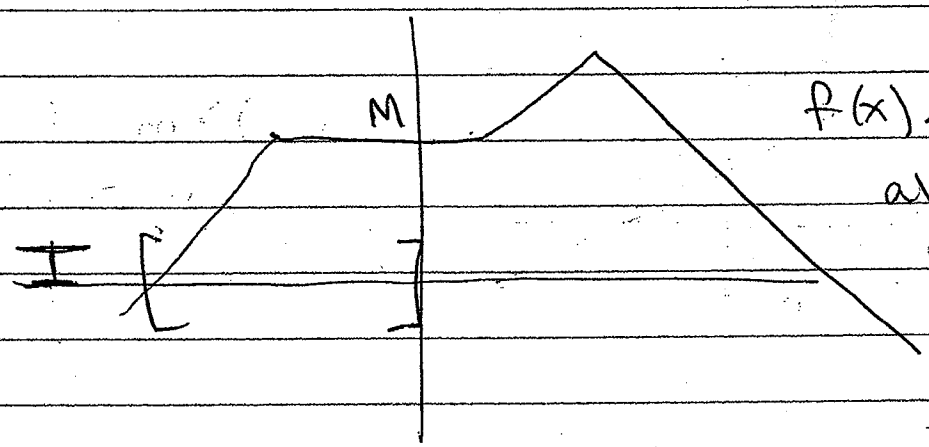
* Other y-values could be equal to M or less than M.



$f(x) \leq M$ for all $x \in I$

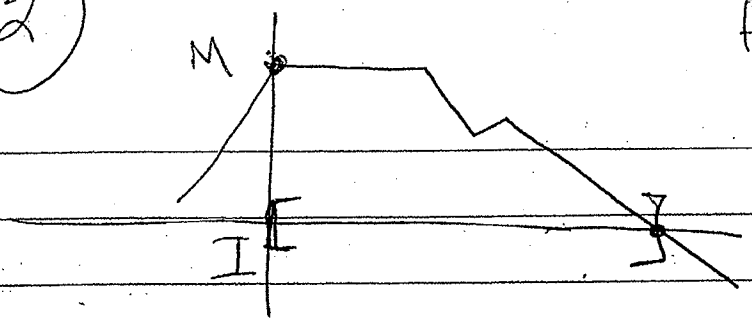


$f(x) < M$ for all $x \in I$



$f(x) \leq M$ for all $x \in I$

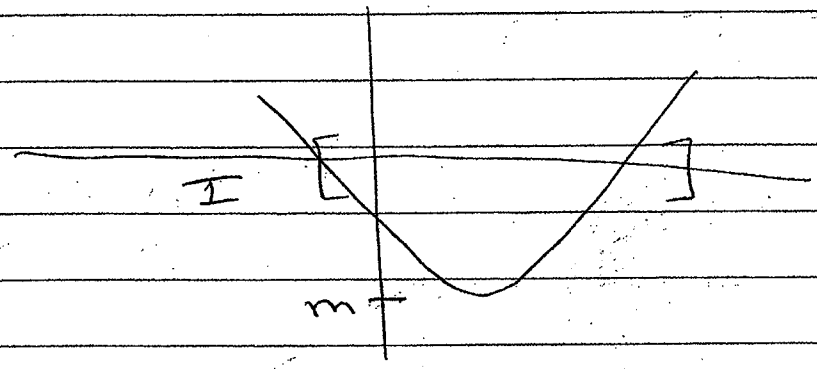
(2)



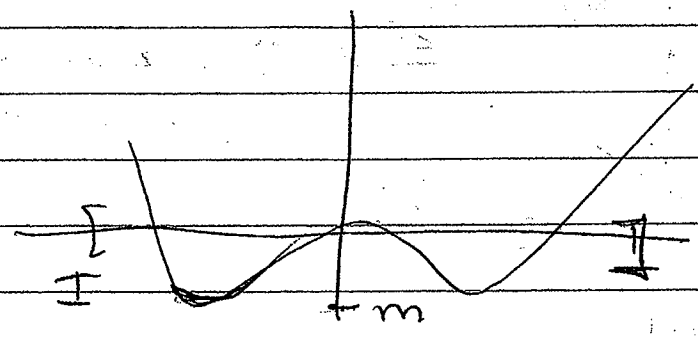
$f(x) \le M$ for all $x \in I$

Also, $f(x) \ge m$ for all $x \in I$

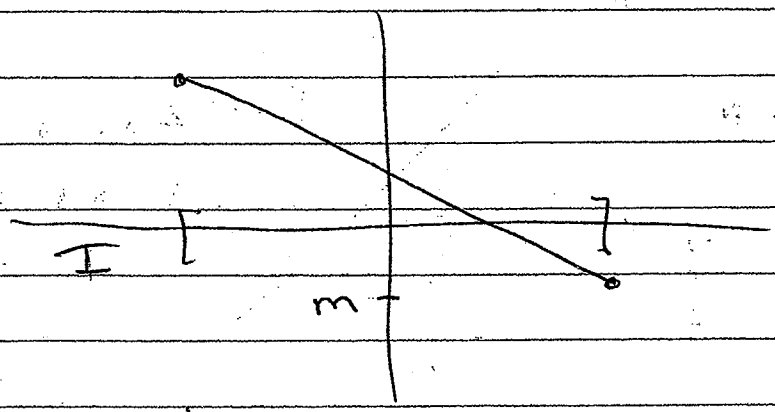
The requirement that m be an <u>absolute minimum</u> on (closed) interval I is that no other y-value of the fcn be less* than m.



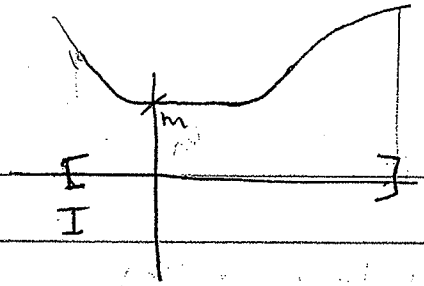
$f(x) > m$ for all $x \in I$



$f(x) \le m$ for all $x \in I$

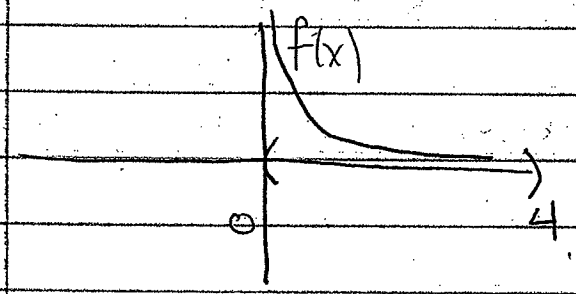


$f(x) > m$ for all $x \in I$

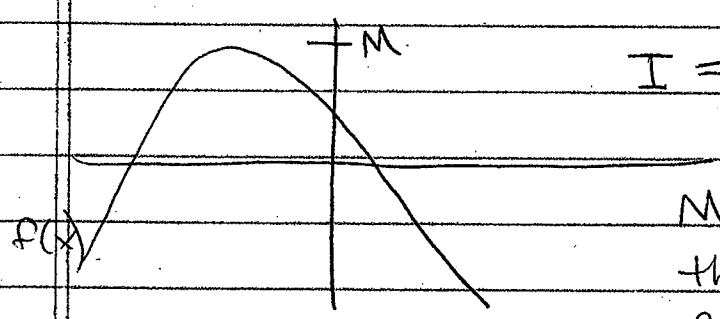


$f(x) \geq m$
for all $x \in I$

What happens when we let interval I be open?
The short answer is that the endpts become an issue. But it's still a valid assumption.



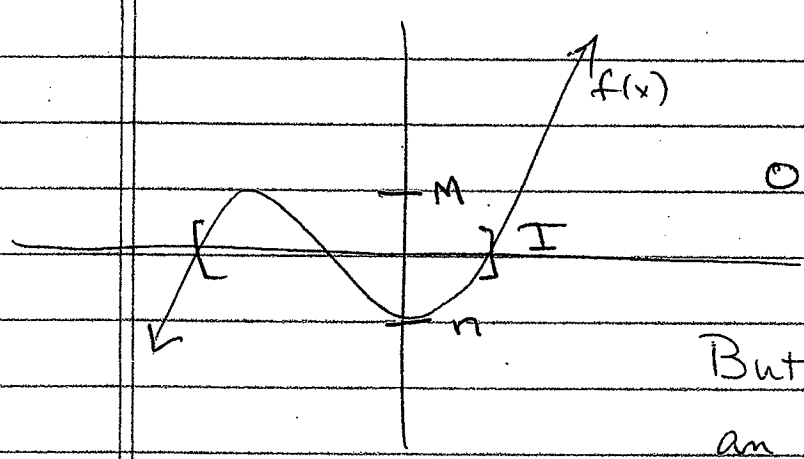
Suppose $I = (0, 4)$ $\longrightarrow$ $f(x) = \frac{1}{x}$ has no abs max M or abs min m in I



$I = \mathbb{R}$ * (the entire number line)

M is the abs max, but there is no abs min because $f(x)$ is unbounded in the negative sense.

* If I = a smaller open interval, will $f(x)$ have an absolute minimum



On I, $f(x)$ has a abs max M and an abs min m.

But on $\mathbb{R}$, it has neither an abs max or min.

(4) The approach to finding the absolute max + min is to check the value of the fcn. at all local extremes + at the endpts, if any.

Eg 22.3 p. 182 is an excellent example

$f(x) = x^3 - 3x$ Make a sketch.

y-int $f(0) = 0$, x-int: $f(x) = x(x^2-3)$

x-int: $0 = x(x^2-3)$

$\boxed{x=0}$, $x^2 - 3 = 0$

$x^2 = 3$

Roots $\boxed{x = \pm\sqrt{3}}$

Crit pts $f'(x) = 3x^2 - 3 = 0$ at $\boxed{x = \pm 1}$

$f''(x) = 6x$, $f''(-1) = -6 < 0$

so $x = -1$ is local max

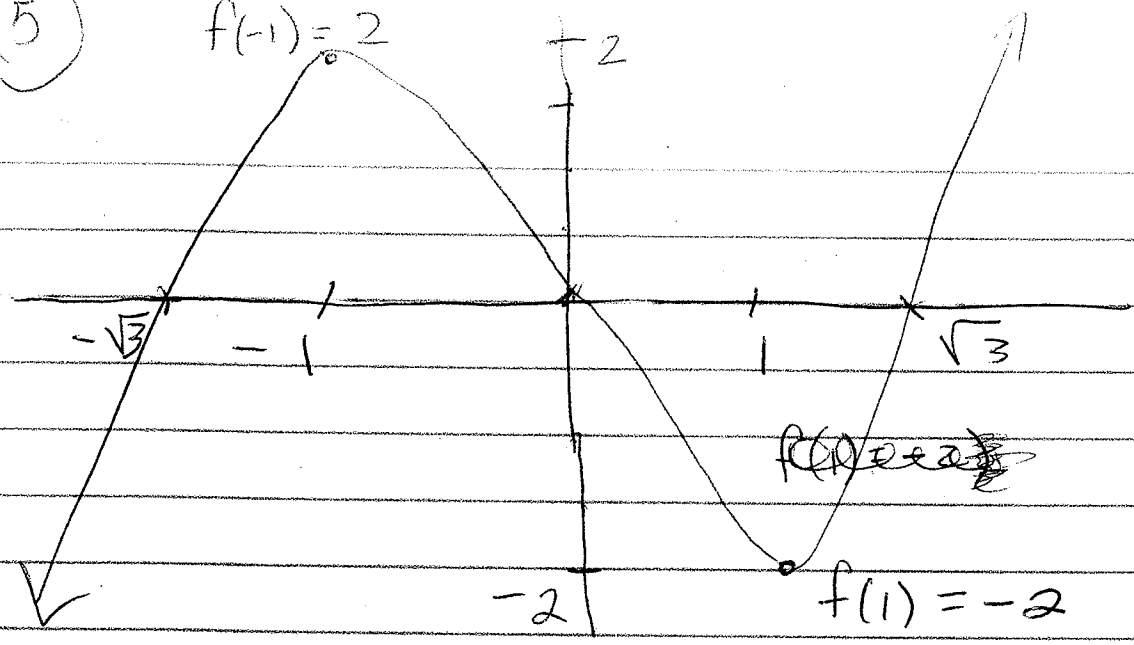
$f''(1) = 6 > 0$ so $x=1$ is local min

POI $f''(x) = 6x = 0$ at $\boxed{x=0}$

By SDT, $f'' < 0$ on $(-\infty, 0)$ c. down

$f'' > 0$ on $(0, \infty)$ c. up

5



For each interval, we inspect for absolute max, min, if they exist:

$(-\infty, \infty)$ No abs. extremes
$(f \to -\infty \text{ \& } +\infty)$

$[-4, 4]$ The extremes occur at the end values, where the fcn is clearly less than $f(1)$ and greater than $f(-1)$.

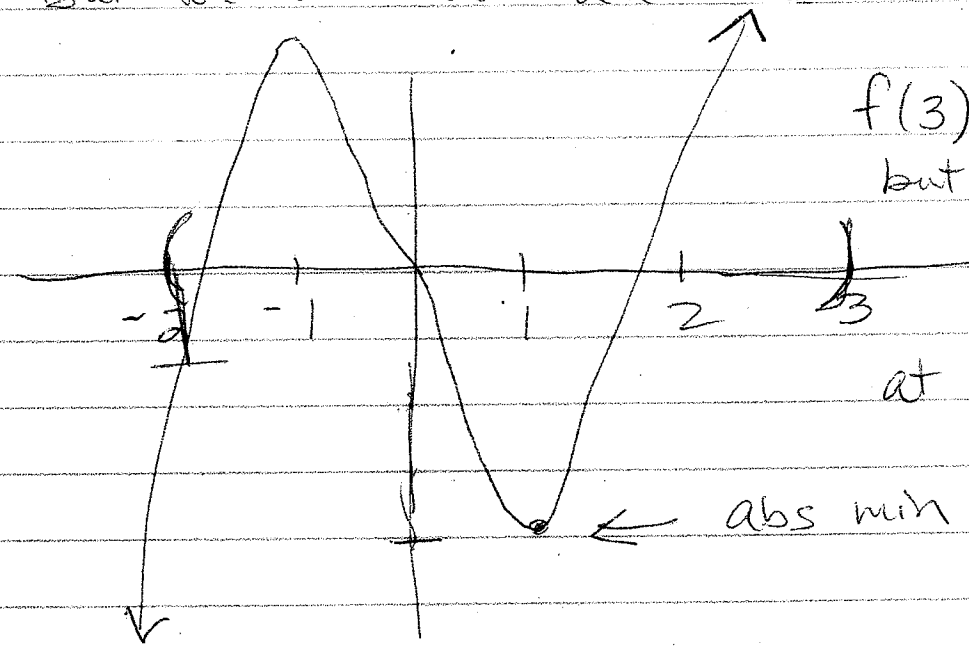
$$f(-4) = (-4)^3 - 3(-4) = -52 < f(1)$$
$$f(4) = 4^3 - 3(4) = 52 > f(-1)$$

$[-2, 2]$ We should check these, though your graph might be clear.

$f(-2) = (-2)^3 - 3(-2) = -2 = f(1)$ same as local min

$f(2) = (2^3) - 3(2) = 2 = f(-1)$ same as local max

(6) Finally, on $(-2,3)$, the interval is open, so we have only the local max, at $x=-1$. But we do have an abs. min, at $x=1$



$f(3)$ is not plotted, but clearly it's greater than $f(-1)$ at values close to $x=3$.

Endpt. analysis on <u>open</u> intervals requires some stronger tools of analysis than this course offers, so we accept an intuitive grasp of abs extremes in these cases.