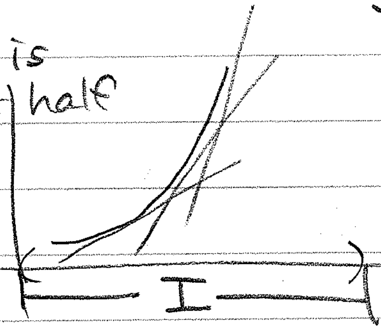
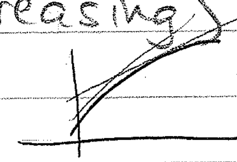
Curve Sketching using calculus

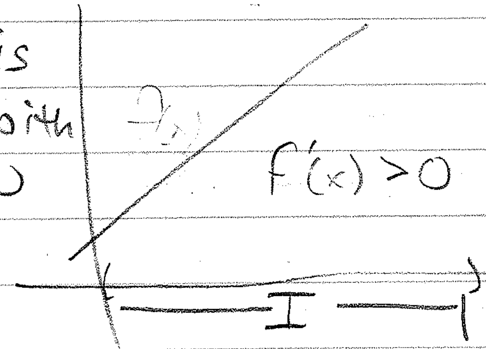
$f(x)$ is right half of x^2



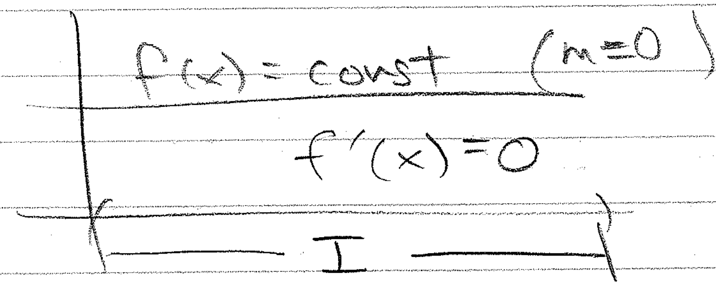
Slope of tangent is positive (and is also increasing) on I



$f(x)$ is line with $m>0$



Slope of tangent is positive on I

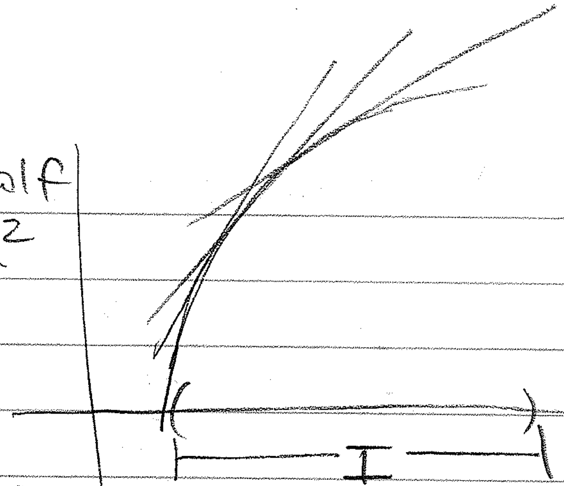


Slope of tangent is zero on I

In the first two cases, f is increasing. Notice that the derivative is positive.

Theorem 1 If I is an open interval throughout which $f'(x)>0$, then $f(x)$ is increasing on I. If $f'(x)<0$ throughout I then $f(x)$ is decreasing on I

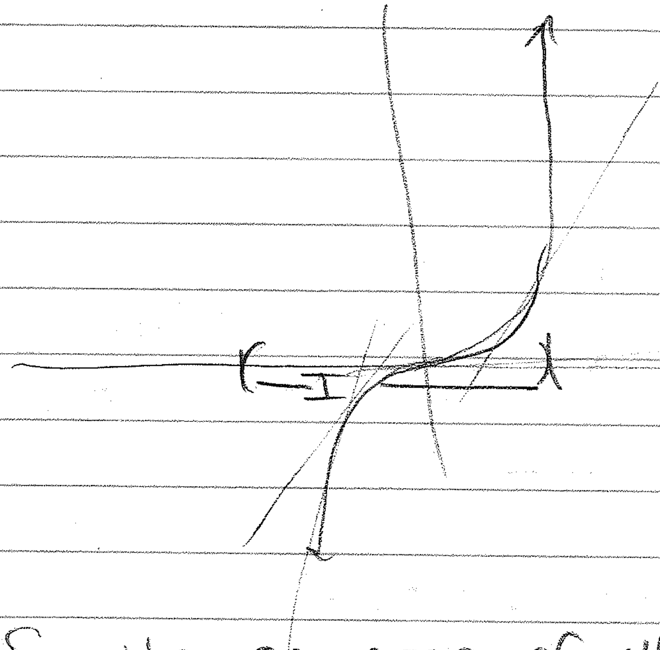
$f(x)$ is left half of $-x^2$



Slope of tangent is positive (but in this case it's getting less steep)

By the theorem, $f(x)$ is increasing on I

Now consider $\underline{f(x) = x^3}$



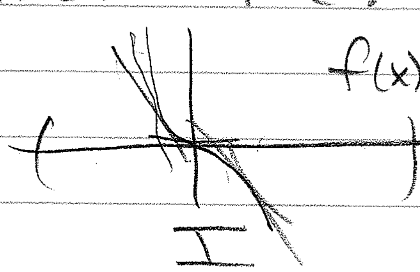
$f'(x)$ is >0, then $=0$, then >0 again on I

This does not reflect the theorem statement, which is a strict inequality of $f(x) > 0$

So, the converse of the theorem is not true,

Thm 2: $\Longrightarrow$ If $f(x)$ is increasing on open interval I and if it's differentiable on I then $f'(x) \geq 0$. If $f(x)$ is decreasing on open interval I and if it's differentiable on I, then $f'(x) \leq 0$.

$f(x) = -x^3$



Logic statements

True ~~statement~~ If A then B $\quad A \to B$

"A implies B"

Contrapositive If not A then not B is also true.

But the converse is not necessarily true.

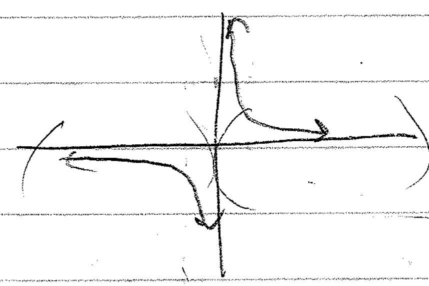
If B then A

Inverse If not A then not B

Ex 17.4 $\quad f(x) = xe^x$

p. 142 #1, 2b, f, i

Ex 17.2 $\quad f(x) = \frac{1}{x}$



$f'(x) = -\frac{1}{x^2} < 0$ for all x on domain (which is two open intervals)

Hence on I_1 $(-\infty, 0)$, f is decreasing and on I_2 $(0, \infty)$ f is also decreasing.

Ex 17.3 $\quad f(x) = \dfrac{x^2-2x+1}{x-3} = \dfrac{(x-1)^2}{x-3}$

$f'(x) = \dfrac{(x-1)(x-5)}{(x-3)^2}$, which $= 0$ at $x = 1, 5$

(Notice $x = 3$ is not a critical number since $x=3$ is not in domain $f(x)$)

Checking x values on either side of 1 + 5

$x=0 \qquad 1 \qquad x=2 \qquad 5 \qquad x=6$

$f'(0) = \dfrac{(-)(-)}{+} > 0$, so f is $\nearrow$ on $(-\infty, 1)$

$f'(2) = \dfrac{(+)(-)}{+} < 0$, so f is $\searrow$ on $(1,5)$

$f'(6) = \dfrac{(+)(+)}{(+)} > 0$, so f is $\nearrow$ on $(5, \infty)$

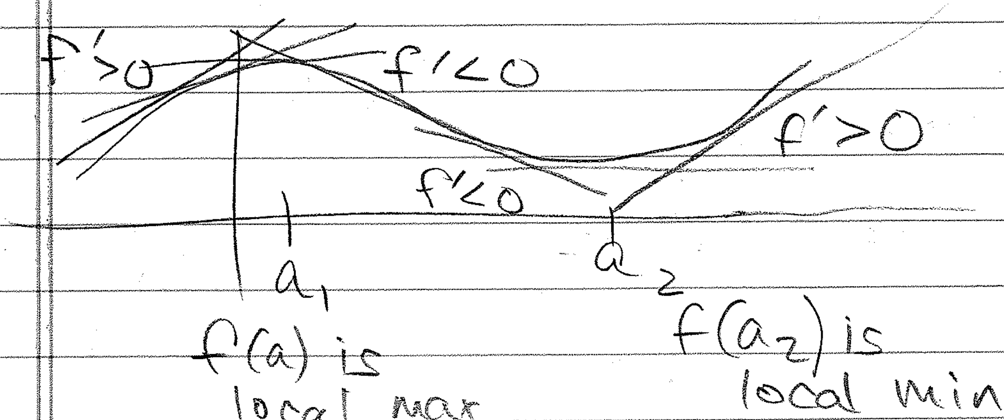
By the way, the reason we know that any value in each interval suffices to show the der is > or < zero in that interval is because of the intermediate value theorem lets us prove no other root of f' exist on that interval.

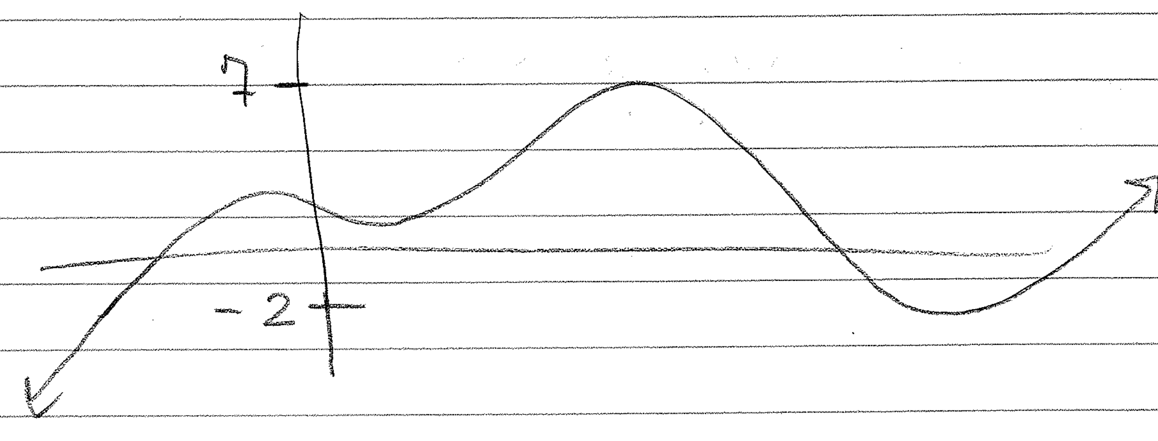
Sec 17 Increasing, decreasing fcns & extrema

First derivative test

If $x=a$ is a critical number of f ($f'(a)=0$ or DNE) and f' changes sign from positive to negative, then f has a local maximum at $x=a$.

If f' changes sign from negative to positive then f has a local minimum at $x=a$





<u>Review</u> End behavior of polynomials

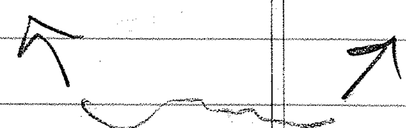
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

y-int $f(0) = a_0$

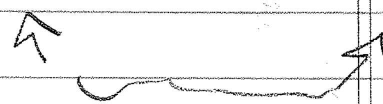
x-int $f(x) = 0$ roots

<u>Ends</u> if n is even and $a_n > 0$

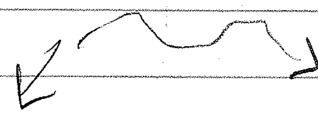
look like this



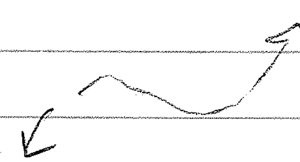
$y = x^2$ n even, $a_n > 0$

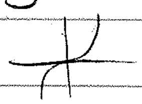


$y = -x^2$ n even, $a_n < 0$

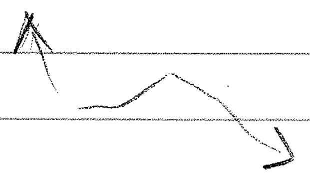


$y = x^3$ n odd, $a_n > 0$



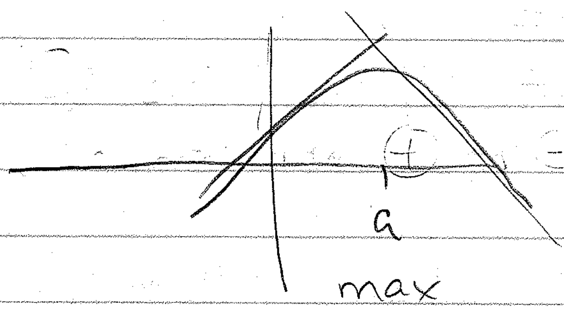


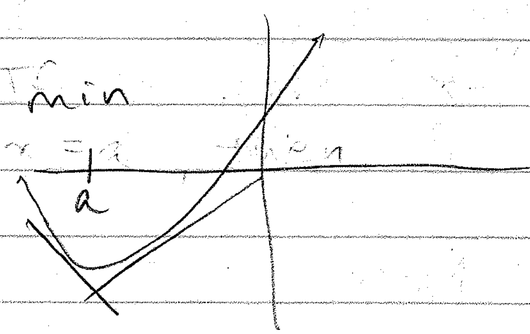
$y = -x^3$ n odd, $a_n < 0$



Sec 18 Concavity + Inflection points

First derivative test – If the sign of f' changes from $(+)$ to $(-)$ at $x=a$, then $f(a)$ is a local max. If f' changes from $(-)$ to $(+)$ at $x=a$, then $f(a)$ is a local min.

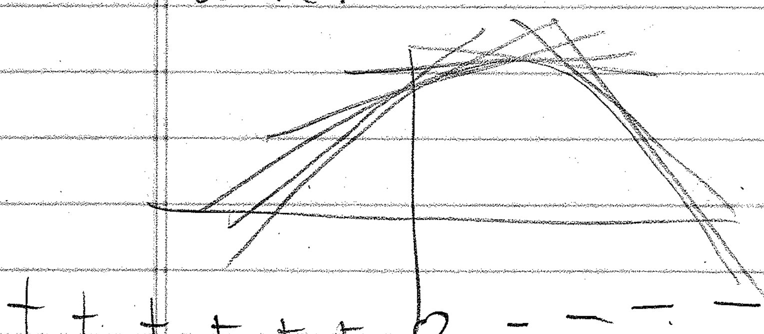


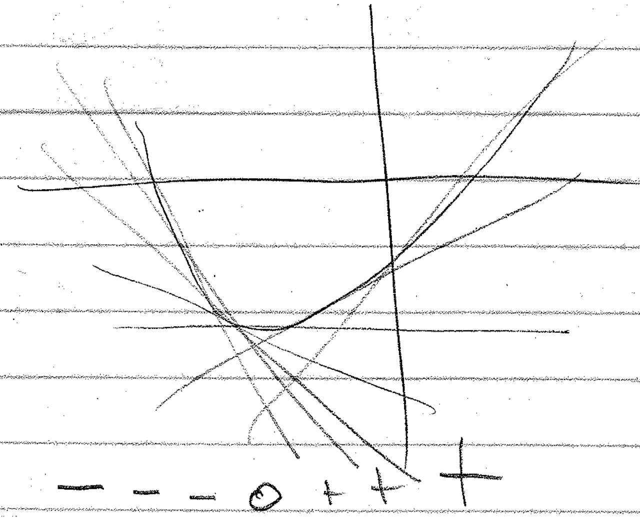


We call the shape of the graph on interval seen "concave down".

We call this shape "concave up"

The trend of the first derivative – its rate of change – is described by the second derivative.





Second derivative test

If $f'(a)=0$ and the graph is concave down at $x=a$, then $f(a)$ is a local max.

If $f'(a)=0$ and the graph is concave up at $x=a$, then $f(a)$ is a local min.

Statement of test: If $f'(a)=0$ and $f''(a)<0$ then $f(a)$ is a local max. If $f'(a)=0$ and $f''(a)>0$, then $f(a)$ is a local min

Note: If $f''(a)=0$ also, then you might still have a local min or max, or an inflection point.

To find out, you could either use the first derivative test or the second derivative test on intervals either side of a.

1st der. test

$\boxed{f'(a)=0}$

	a		
$f'>0$		$f'<0$	$\longrightarrow$ f(a) is local min
$f'<0$		$f'>0$	$\longrightarrow$ f(a) is local min
$f'>0$		$f'>0$	$\longrightarrow$ f(a) inflection pt

2nd der. test

$\boxed{f''(a)=0}$

	a		
$f''>0$		$f''<0$	$\longrightarrow$ inflection at $x=a$
$f''>0$		$f''>0$	$\longrightarrow$ Concave up at $x=a$
$f''<0$		$f''<0$	$\longrightarrow$ concave down at $x=a$