To know for Test 3

Ch 7

- The solution to $f(x) \geq$ or $f(x) \leq 0$ requires finding zeroes to $f(x)$ and doing a sign analysis.

- The solution to $\dfrac{f(x)}{g(x)} \geq 0$ or $\dfrac{f(x)}{g(x)} \leq 0$ requires finding the zeroes of $f(x)$ and $g(x)$, then doing a sign analysis of the intervals formed and the zeroes themselves.

- The solution to $\dfrac{f(x)}{g(x)} \geq c$ requires bringing c to the left, $\dfrac{f(x)}{g(x)} - c \geq 0$,

  finding the LCD and doing a sign analysis on the zeroes.

- The solution to $|f(x)| = a$ is $f(x) = a$ or $f(x) = -a$.

- The solution to $|f(x)| < a$ is $-a < f(x) < a$.

- The solution to $|f(x)| > a$ is $f(x) > a$ or $f(x) < -a$.

- The solution to $|f(x)| = |g(x)|$ requires looking at the cases $f(x) = \pm g(x)$ or by squaring both sides, as long as that doesn't give a more complicated answer.

- If you have an absolute value on both sides with a constant added, but squaring makes the problem too complicated, then do it like Problem #5h in Sec. 7.2. The plan is as follows:

  Given $|f(x)| = |g(x)| + c$. Look at the cases below, and for each, inspect whether the solution is in respective domain:

  $f(x) = g(x) + c$ when $f(x) > 0$ and $g(x) > 0$;
  $f(x) = -g(x) + c$ when $f(x) > 0$ and $g(x) < 0$;
  $-f(x) = -g(x) + c$ when $f(x) < 0$ and $g(x) < 0$;
  $-f(x) = g(x) + c$ when $f(x) < 0$ and $g(x) > 0$.

- The solution to $|f(x)| > |g(x)|$ is usually best solved by squaring both sides. Then proceed as usual.

Ch 10

Know your properties of exponential functions and logarithms. Both in essence are derived from the rules of exponents, which you learned early on. We had plenty of handouts on these. Summarize the important properties below:

$x^m x^n = x^{m+n}$ $\frac{x^m}{x^n} = x^{m-n}$ $\left(x^m\right)^r = x^{mr}$ $x^{\log_b x} = x$

$\log_b(xy) = \log_b x + \log_b y$ $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ $\log_b x^r = r\log_b x$

$\log_e x \equiv \ln x$ $\log_{10} x \equiv \log x$

$\ln e = 1$ $\log 10 = 1$

Change of base property: $\log_b a = \frac{\log_c a}{\log_c b}$. (To change a $\log_b a$ to a common or natural log, generally.)

If $x = y$ then $\log x = \log y$. If $x = y$ then $a^x = a^y$.

$a^{\log_a x} = x$ $\log_a a^x = x$

Exponential and logarithmic functions are inverse functions.

Domain of $y = \log x$ is $x > 0$. Range is $y \in \mathbb{R}$.

Domain of $y = a^x$ is $x \in \mathbb{R}$. Range is $y > 0$.

Ch 2.4

Sigma notion: Know the following closed form solutions to the following sums:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n \qquad \text{e.g.,} \quad \sum_{i=1}^{5}(2x_i - 3) = (2x_1 - 3) + (2x_2 - 3) + (2x_3 - 3) + (2x_4 - 3) + (2x_5 - 3)$$

$$\sum_{i=1}^{n} c = nc \qquad \text{e.g.,} \quad \sum_{i=1}^{8} 4 = 8(4) = 32$$

$$\sum_{i=m}^{n} c = (n - m + 1)c \qquad \text{e.g.,} \quad \sum_{i=5}^{19} 1 = (19 - 5 + 1)(1) = 15$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{e.g.,} \quad \sum_{i=1}^{40} i = \frac{40(40+1)}{2} = 20(41) = 820$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \text{e.g.,} \quad \sum_{i=1}^{9} i^2 = \frac{9(10)(19)}{6} = \frac{1080}{6} = 160$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \qquad \text{e.g.,} \quad \sum_{i=1}^{4} i^3 = \frac{16(25)}{4} = \frac{400}{4} = 100$$

You can break up series as follows, for example:

$$\sum_{i=1}^{20}(4i+3) = 4\sum_{i=1}^{20} i + \sum_{i=1}^{20} 3 = 4\left[\frac{20(21)}{2}\right] + 20(3) = 840 + 60 = 900$$

To find a series starting at $i = m > 1$, subtract the series parts: $\sum_{i=m}^{n} a_i = \sum_{i=1}^{n} a_i - \sum_{i=1}^{m-1} a_i$. For example:

$$\sum_{i=4}^{16} 5i = \sum_{i=1}^{16} 5i - \sum_{i=1}^{3} 5i = 5\left[\sum_{i=1}^{16} i - \sum_{i=1}^{3} i\right] = 5\left[\frac{16(17)}{2} - \frac{3(4)}{2}\right] = 5[8(17) - 3(2)] = 5(30) = 140$$