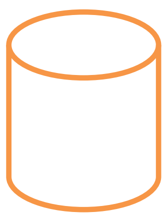
MATH 220 QUIZ 7 DUE THURSDAY MARCH 27 SHOW ALL WORK

NAME____

1. *Minimizing cost.* A *closed-top* cylindrical container is to have a volume of 250 in^2. Assume the costs of the materials for making the cylindrical container described are \$0.02/sq. in. for the ends and \$0.01/sq. in. for the side.

a) Draw a diagram.



b) Write the constraint and objective functions.

Constraint: $V = 250 \text{ in.}^2 = \pi r^2 h$, Objective fcn: $SA = 2\pi rh + 2\pi r^2$

So, the cost that is to be minimized is given as: $SA\ cost = (0.01)2\pi rh + (.02)2\pi r^2$

Reduce *SA cost* to one variable, so substitute $250 \text{ in.}^2 = \pi r^2 h$, solved for h in terms of r

c) Set up the correct single-variable function to minimize.

$$h = \frac{250}{\pi r^2} \quad \text{into} \quad SA\ cost = (0.01)2\pi r\frac{250}{\pi r^2} + (.02)2\pi r^2$$

d) Find the dimensions r and h that *minimize cost* of producing this container.

First, simplify your objective function:

$$SA\ cost = C(r) = (0.01)2\pi r\frac{250}{\pi r^2} + (.02)2\pi r^2 = 0.02\frac{250}{r} + 0.04\pi r^2 = \frac{5}{r} + 0.04\pi r^2$$

Now, take the derivative and set equal to 0:

$$C'(r) = \frac{-5}{r^2} + 0.08\pi r = 0 \qquad \frac{5}{r^2} = 0.08\pi r \qquad r^3 = \frac{5}{.08\pi}$$

So, $r = \sqrt[3]{\dfrac{5}{.08\pi}}$ inch and $h = \dfrac{250}{\pi\left(\sqrt[3]{\dfrac{5}{.08\pi}}\right)^2}$ or $\dfrac{250}{\pi\left(\dfrac{5}{.08\pi}\right)^{2/3}}$ inch

I didn't bother to do the calculator computation on this, and you won't on the exam.

2. *Determining ticket price.* Promoters of international fund-raising concerts walk a fine line between profit and loss when determining ticket price for closed-circuit TV showings in local theaters. A theater determines that at an admission price of \$26, an average of 1000 people will attend an event. For decrease drop in price of \$1, the theatre gains 50 customers.

[Before you begin, note, I'm asking you to use the approach in Prof. McKinney's video. If you want to check your work with the $R(n)$ method in the slides and some of the readings, that's fine. But you must answer the following by determining $R(p)$.]

a) Find a linear equation that gives the number of tickets sold q at price p.

We have two points on $q(p)$, the first being (26, 1000), and the second (25, 1050). Hence, $m = \dfrac{1000-1050}{26-25} = -50$. Using point-slope form of the line:

$$q - q_1 = m(p - p_1) = q - 1000 = -50(p - 26)\text{ , or } q = -50p + 2300$$

b) Find $R(p)$, the *revenue* from ticket sales as a function of the price per ticket.

$$R(p) = pq = p(-50p + 2300) = -50p^2 + 2300p$$

c) Find the ticket *price* that maximizes revenue from ticket sales. Show all work.

$$R'(p) = -100p + 2300 = 0\text{, so } p = \$23$$

By either FDT or SDT, we find this is a maximum.

FDT: $R'(0) > 0$, $R'(25) < 0$, $R(23)$ is local max.

SDT: $R''(p) = -100p$, $R''(23) = -2300 < 0$, so $p = 23$ is in c. d. interval, and hence a local max.

d) Finally, find the maximum possible revenue from ticket sales.

$$R(23) = -50(23)^2 + 2300(23) = \$26{,}450$$