Optimization: Focus problems from each category

Category 1: Basic (single function) optimization (taken from Supplemental Materials)

1. The cost to manufacture x units of a product is $15000 + 40x + 0.02x^2$. The revenue from the sale of these products is $100x - .01x^2$.

Find the level of sales that will maximize profit.

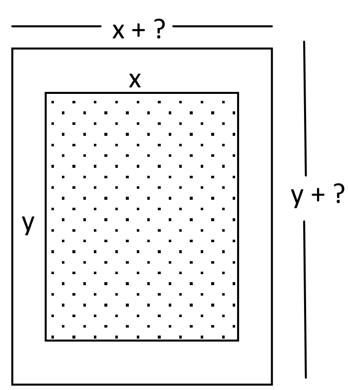
2. A manufacturer makes a product at a cost of \$12/unit. He estimates demand will be $50 - p$ units when the price is p dollars (for example, at \$10, he can sell 40 units). Find the cost, revenue and profit equations, as functions of price p. Then find the price that maximizes the profit.

3. A manufacturer builds q items at a cost of $6000 + 5q + 0.01q^2$. To sell q items, the price needs to be $20 - \frac{q}{4}$. (So these are your C and p functions.) Find the quantity that will maximize profit.

Category 2*: Geometry (two functions, a constraint and a function to be optimized)*

4. A printer receives an order to produce a rectangular poster containing 648 square centimeters of print surrounded by margins of 2 cm on each side and 4 cm on the top and bottom. What are the dimensions of the smallest piece of paper that can be used to make the poster?

Hint: Look at the figure. Let the rectangle with print have dimensions x by y. Its area gives the constraint equation. Now, express the dimensions of the larger rectangle in terms of x and y by adding the 2 cm and 4 cm appropriately. This rectangle's area is your function to be minimized.



5. A cylindrical container with no top is to be constructed to hold a fixed volume of liquid. The cost of the material used for the bottom is 3 cents per square inch, and that for the curved side is 2 cents per square inch. Use calculus to derive a simple relationship between the radius and height of the least expensive container.

6. (For fun) Use the fact that 12 fluid ounces is approximately 6.89 cubic inches to find the dimensions of the 12-ounce soda can that can be constructed using the *least* amount of metal (and hence the least materials cost). But these are not the dimensions of your typical can of soda. Measure a can if one is handy. Compare these dimensions with the answer to this problem. What do you think accounts for the difference?

Category 3: Hot dog problems (n = # of price reductions or increases; or linear modeling)

7. A tool company determines it can achieve 500 daily rentals of jackhammers per year at a daily rental fee of $30. Each $1 increase in rental fee results in 10 fewer jackhammers being rented. Find the rental price that maximizes revenue.

8. An apple farm yields an average of 30 bushels of apples per tree when 20 trees are planted on an acre of ground. Each time 1 more tree is planted per acre. The yield is decreased by 1 bushel per tree, as a result of crowding. How many trees should be planted on an acre in order to get the highest yield?

9. Finally, a hot dog problem. A ball park vendor finds it can sell 10,000 hot dogs if they charge $4 for each hot dog. They have found that raising the price 25 cents per hot dog, they sell 500 fewer hot dogs. Determine the price they should sell the hot dogs at to maximize revenue. DO THIS BY USING BOTH METHODS:

Method a) Let n = # of 25-cent price increases, so $R(n) = p(n)\cdot x(n)$. Solve for n by maximizing $R(n)$. Proceed as in class to find n, then $p(n)$.

Method b) On the (x, p)-plane, Point 1 is (10,000, $4), Point 2 is (?, ?). Find slope m, then the demand equation $p(x) = mx + b$. Finally, $R(x) = x\cdot p(x)$. Maximize this revenue function to solve for x. Plug x into $p(x)$. You should get the same answer as in the first method!