Practice Quiz 1 for Integration Quiz 1

1. Find average value of $f(x) = \ln x$ on $[1, e^2]$

2. Find the area between $f(x) = \sqrt{x}$ and $g(x) = 1$ on $[0, 2]$.

3. Raggs, Ltd., determines its marginal cost each week is given by $\boxed{C'(x) = -\frac{2}{25}x + 50}$ for $\boxed{x \le 450 \text{ dresses}}$.
domain
Find total cost of producing the first 200 dresses.

1.
$$\frac{1}{b-a}\int_a^b \ln(x)\,dx = \frac{1}{e^2-1}\int_1^{e^2} \ln x\,dx$$

$$= \frac{1}{e^2-1}\Big[x\ln x - x\Big]_1^{e^2}$$

$$= \frac{1}{e^2-1}\Big[e^2 \underbrace{\ln e^2}_{2\ln e} - e^2 - 1\cancel{\ln 1} + 1\Big]$$

$$= \frac{1}{e^2-1}\Big[2e^2 - 1e^2 + 1\Big] = \boxed{\frac{e^2+1}{e^2-1}} = y = f(c)$$

$c \in [1, e^2]$

not an area, it's a height $f(c)$

$$\underline{\ln c} = \frac{e^2+1}{e^2-1}$$

Def of $\ln x = y \Rightarrow e^y = x$

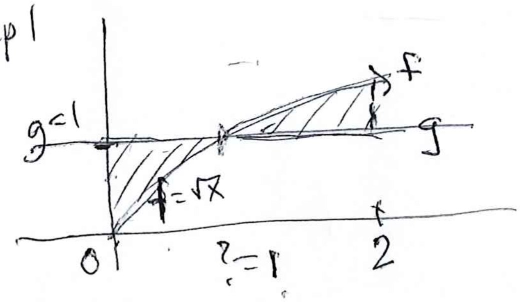
$$c = e^{\frac{e^2+1}{e^2-1}}$$

Here is where f attains its avg value on $[1, e^2]$

2.)

Step 2

Step 1



$\sqrt{x}^2 = 1^2$ to find intersection

$x = 1$ $\int$ upper − lower fcn

Step 3 $\displaystyle\int_0^1 1-\sqrt{x}\,dx + \int_1^2 \sqrt{x}-1\,dx$

Step 4 $\displaystyle x - \frac{x^{3/2}}{3/2}\Big|_0^1 + \frac{x^{3/2}}{3/2} - x\Big|_1^2$

Step 5 $\displaystyle\left(1-\frac{1}{3/2}\right) - \cancel{(0-0)} + \left(\frac{2^{3/2}}{3/2} - 2\right) - \left(\frac{1^{3/2}}{3/2} - 1\right)$

$\displaystyle = \cancel{1} - 2/3 + \frac{2}{3}\sqrt{8} - \cancel{2} - \cancel{\frac{2}{3}} + \cancel{1}$

$\displaystyle = -\frac{4}{3} + \frac{2\cdot 2\sqrt{2}}{3} = \boxed{\frac{4\sqrt{2}-4}{3}}$

(3) total fcn $= \displaystyle\int_a^b$ rate fcn "accumulation"
value
quantity

$C'(x) = -\frac{2}{25}x + 50$ weekly,

$\displaystyle C_{TOT} = \int_0^{200} -\frac{2}{25}x + 50\,dx$

$\displaystyle = -\frac{\cancel{2}x^2}{\cancel{2}\cdot 25} + 50x\Big|_0^{200} = \frac{-40000}{25} + 50(200)$

40000

-0

$= -1600 + 10{,}000 = \$8400$