Practice Quiz

1. $\int m x^1 \, dx = \frac{mx^2}{2} + C$

2. $\int \frac{x^2}{7} dx = \frac{1}{21}\left(x^3 + C\right) = \frac{x^3}{21} + C$

3. $\int \left(\frac{2}{x} - \sqrt{x} - 6\right) dx = 2 \ln x - \frac{x^{3/2}}{3/2} - 6x + C$

$x > 0$

4. $\int x \sqrt{x+2} \, dx = \int x \sqrt{u} \, du = \int (u-2) u^{1/2} du$

Let $\boxed{u = x+2}$

$du = dx$

$x = u - 2$

need all u

$= \int u^{3/2} - 2u^{1/2} \, du$

$\boxed{\frac{u^{5/2}}{5/2} - \frac{2u^{3/2}}{3/2} + C}$ $\Leftarrow$ $\boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C}$

Now do this with bounds of $\int$ $a = 0$, $b = 2$ (x values)

$\int_{x=0}^{2} x\sqrt{x+2} \, dx = \int_{u=2}^{4} u^{3/2} - 2u^{1/2} \, du$

$u = x + 2$

$u = 0 + 2 = 2$

$u = 2 + 2 = 4$

Substitute new bounds into the u-sub soln:

$\left[\frac{2}{5}\left(4^{5/2}\right) - \frac{4}{3}\left(4^{3/2}\right)\right] - \left[\frac{2}{5}\left(2^{5/2}\right) - \frac{4}{3}\left(2^{3/2}\right)\right] = \ldots$

$F(b) - F(a)$

5. $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$\int u\,du = \frac{u^2}{2} + C$

$= \boxed{\frac{(\ln x)^2}{2} + C}$

6. $\int xe^{-x}\,dx = -xe^{-x} - \int -e^{-x}\,dx$

uv

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$= -xe^{-x} + \int e^{-x}\,dx = \boxed{-xe^{-x} - e^{-x} + C}$

Suppose bounds $a = 0$, $b = 1$:

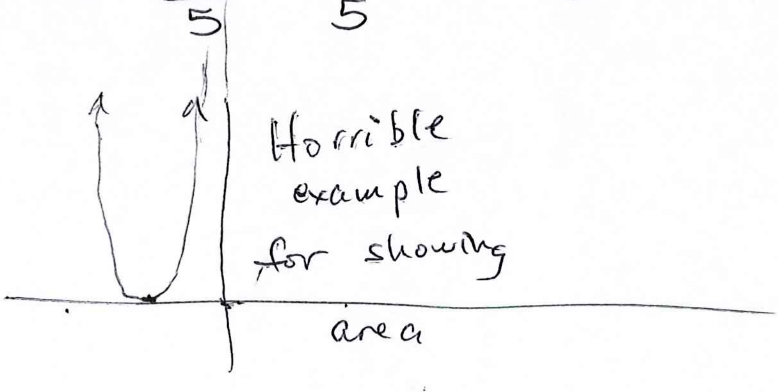
$= -xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx = \left(-xe^{-x} - e^{-x}\right)\Big|_0^1 = ?$

$= \left(-1e^{-1} - e^{-1}\right) - \left(0 - e^{0}\right) = -e^{-1} - e^{-1} + 1 = \boxed{-\frac{2}{e} + 1}$

7. $\int_{x=0}^{2} (x+1)^4\,dx$, $u = x+1$, $du = dx$

$\int_{u=1}^{3} u^4\,du = \frac{u^5}{5}\Big|_1^3$

$= \frac{3^5}{5} - \frac{1}{5} = \frac{80}{5} = \boxed{16}$



Horrible example for showing area

8. $\displaystyle\int_1^e \frac{5}{x}\,dx = 5\ln x \Big|_1^e = 5(\ln e - \ln 1) = 5(1-0) = 5$

9. $\displaystyle\int_{x=0}^{1} \frac{x}{x^2+1}\,dx$ let $u = x^2+1$, $du = 2x\,dx$, $\frac{du}{2} = x\,dx$ $\displaystyle\frac{1}{2}\int_{u=1}^{u=2} \frac{du}{u} = \frac{1}{2}\ln u \Big|_1^2$

$= \frac{1}{2}(\ln 2 - \ln 1) = \boxed{\frac{1}{2}\ln 2}$

10. $\displaystyle\int \ln x\,dx = x\ln x - x + C$ via IBP

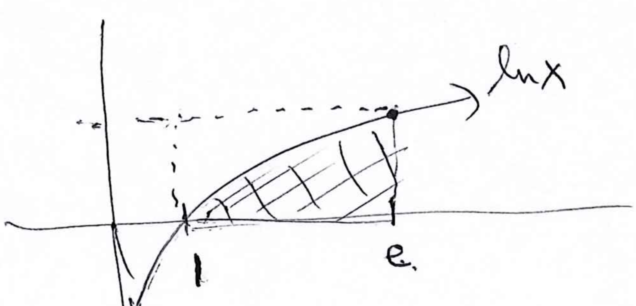
$u = \ln x$ $dx = dv$ $\displaystyle\int \ln x\,dx = x\ln x - \int \cancel{x}\,\frac{dx}{\cancel{x}}$

$du = \frac{dx}{x}$ $v = x$

$= \boxed{x\ln x - x + C}$ //

Show $\displaystyle\int_1^e \ln x\,dx = 1$: $x\ln x - x \Big|_1^e = (e\ln e - e) - (1\ln 1 - 1)$

$= 0 - (0-1) = \boxed{1}$

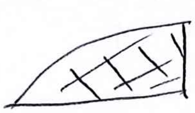


rect base $= e - 1$

height $= \ln e = 1$

Area rect $= (e-1)(1)$

$= 1.7$

 $= 1$