

Sec 7.2 + 7.3 Absolute Value

Equations + Inequalities

From the def of $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

We consider several varieties of abs. value equations:

$$|f(x)| = c, \quad |f(x)| = g(x), \quad |f(x)| \leq |g(x)|$$

$$\text{and } |f(x)| = |g(x)| + c$$

The first type is easy to solve; following the def of $|x|$, we write:

$$|f(x)| = \begin{cases} f(x), & \text{if } f(x) \geq 0 \\ -f(x), & \text{if } f(x) < 0 \end{cases}$$

When $f(x)$ is a few other than just x , proceed as follows:

$$\text{Ex 1. } |2x-3| = 9$$

Case 1 $2x-3 = 9$, when $2x-3 \geq 0$, i.e. $x \geq \frac{3}{2}$

Case 2 $-(2x-3) = 9$ (that is, $2x-3 = -9$) when $2x-3 < 0$
i.e. $x < \frac{3}{2}$

Solving: $2x = 12$, $\boxed{x = 6}$ (case 1)

And $-2x+3=9$, $\boxed{x = -3}$

Notice that this is the same as

writing $|2x-3| = 9$ as

$$2x-3 = 9 - 1 \text{ or } 2x-3 = -9$$

While this gives the correct answer by definition of $|f(x)|$, we're actually solving $2x-3 = 9$ or $-(2x-3) = 9$.

As for the domains, we see our solns are valid:

$$x = 6 \quad \text{when} \quad x > 3/2$$

$$\text{and } x = -3 \quad \text{when} \quad x < 3/2$$

Seems silly to mention domains, since solns clearly lie in intervals named.

However, when we get $|f(x)| = g(x)$ we must be more careful in the set up to follow the definition.

$$\underline{\text{Ex.2}} \quad |3x-5| = x+10$$

By def: On the left,

$$|3x-5| = \begin{cases} 3x-5, & \text{if } 3x-5 \geq 0 \\ -(3x-5), & \text{if } 3x-5 < 0 \end{cases}$$

Simplifying the domain considerations:

$$3x - 5 \geq 0 \rightarrow x \geq 5/3$$

$$3x - 5 \leq 0 \rightarrow x \leq 5/3$$

Now, setting up the cases, solve + check against the domain:

$$|3x - 5| = x + 10$$

Case 1 $3x - 5 = x + 10$ if $x \geq 5/3$

Case 2 $-(3x - 5) = x + 10$ if $x \leq 5/3$

Solving:

Case 1: $3x = 15, x = 15/2$

Case 2: $-3x + 5 = x + 10$

$$-4x = 5, x = -5/4$$

Is $x = 15/2$ in domain $x \geq 5/3$? Yes

Is $x = -5/4$ in domain $x \leq 5/3$? Sure!

So we still have not seen a pressing need to check the domain.

Next type of problem:

$$\text{Ex. 3} \quad \left| \frac{x}{2} - 7 \right| = |x + 4|$$

This one has 4 cases. But as we saw, we can use the property

$$|a|^2 = a^2 \text{ when there is}$$

+ abs value — and only abs value —
fns set equal, to clear the
fences. It gives us a break
again from the domain check:

$$\left| \frac{x}{2} - 7 \right|^2 = |x + 4|^2$$

$$\frac{x^2}{4} - \frac{14}{2}x + 49 = x^2 + 8x + 16$$

$$-\frac{3}{4}x^2 - 15x + 33 = 0$$

$$\textcircled{X} \text{ by } -\frac{4}{3}: \quad x^2 + 20x - 44 = 0$$

This doesn't factor and the QF is messy, but here goes:

$$x = \frac{-20 \pm \sqrt{400 - 4(1)(-44)}}{2}$$

$$= \frac{-20 \pm \sqrt{576}}{2} = \frac{-20 \pm 24}{2}$$

$$x = 2, -11 \quad \text{possible solns.}$$

Checking:

$$x=2 : \quad \left| \frac{x}{2} - 7 \right| = |x+4|$$

$$\left| -6 \right| = |6| \quad \checkmark$$

$$x=-11 \quad \left| \frac{-11}{2} - 7 \right| = \left| -\frac{11}{2} + 4 \right|$$

$$\left| -\frac{25}{2} \right| \neq \left| -\frac{3}{2} \right| \quad \text{no!}$$

Solution: $x=2$

If you'd done this without squaring,
you'd have four cases:

$$\left| \frac{x}{2} - 7 \right| = |x+4|$$

$$\textcircled{1} \quad \frac{x}{2} - 7 = x+4 \quad \text{if } \frac{x}{2} - 7 \geq 0$$

$$\text{and } x+4 \geq 0$$

$$\textcircled{2} \quad \frac{x}{2} - 7 = -(x+4) \quad \text{if } \frac{x}{2} - 7 \geq 0$$

$$\text{and } x+4 < 0$$

$$\textcircled{3} \quad -\left(\frac{x}{2} - 7\right) = x+4 \quad \text{if } \frac{x}{2} - 7 < 0$$

$$\text{and } x+4 \geq 0$$

$$\textcircled{4} \quad -\left(\frac{x}{2} - 7\right) = -(x+4) \quad \text{if } \frac{x}{2} - 7 < 0$$

$$\text{and } x+4 < 0$$

Eqads! I see why they don't give details in the text, but here I will point out that the four cases of $|f(x)| = |g(x)|$:

$$\begin{aligned} f &= g \\ f &= -g \\ -f &= g \\ -f &= -g \end{aligned}$$

reduce to two cases

$$f = g \text{ and } f = -g$$

And the domains resolve themselves when we check our solutions.

Finally, from the text.

Ex 4 from the text:

$$|x-5| = |2x+6| - 1$$

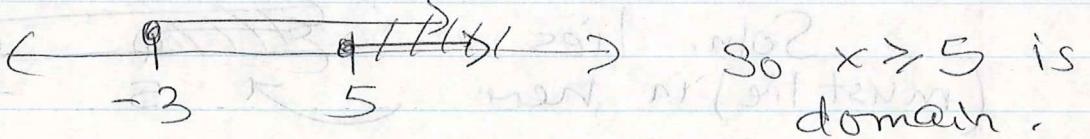
* Squanks We must* write all four cases with their domains and inspect the solns.

work Here's the step-by-step:

- (A) State the cases with their domains
- (B) Simplify the domains and graph them on # line to find where they intersect

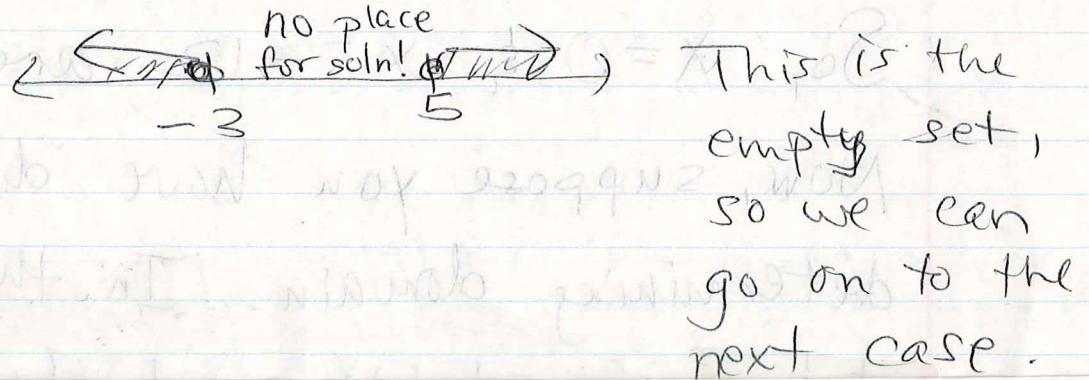
- (C) Solve each case
- (D) Check to see if each soln. lies in the respective domain interval (that's why I use a ~~#~~ line).

Case 1: $x-5 = 2x+6 - 1$ if $x-5 \geq 0$ and $2x+6 \geq 0$
~~so~~ i.e., $x \geq 5$ and $x \geq -3$

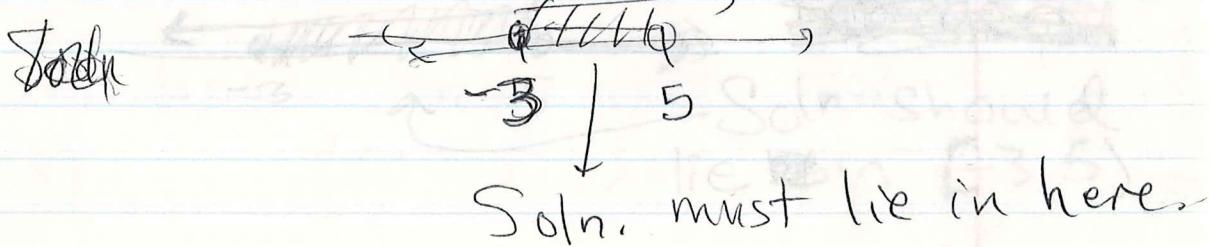


Solving: Case 1, $-x = 10$, $x = 10$
and that's ^{not} good, since $x = 10$ is not in the domain. So discard it.

Case 2: $x-5 = -(2x+6)-1$ if $x > 5$ and $x < -3$



Case 3 $-(x-5) = (2x+6)-1$ if $x < 5$ and $x \geq -3$



Solving: $-(x-5) = (2x+6) - 1$

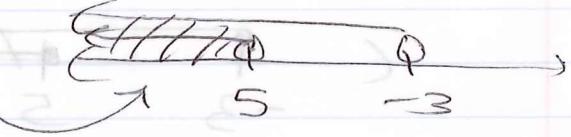
$$-x+5 = 2x+5$$

$$\boxed{x=0}$$

which is in the interval

Case 4 $-(x-5) = -(2x+6) - 1$ if $x < 5$ and $x < -3$

Soln. lies
(must lie) in here



Solving: $-x-(x-5) = -(2x+6)-1$

$$-x+5 = -2x-6-1$$

$$\boxed{x=-12}$$

which lies
in domain

So, $x=0$ & $x=-12$ are the solns.

Now, suppose you have difficulty determining domain. In that event,

take each solution and check it into the original problem. You'll see

that two work and two don't.

$$(x=0, x=-12)$$

$$(x=-10, x=-1)$$

We never found one of the four because we had an inconsistent domain. (Case 2).

Let's do it now, w/o looking at domain.

$$x-5 = -(2x+6) - 1$$

$$x-5 = -2x-6 - 1$$

$$\rightarrow x = -2$$

$x = 2 \rightarrow$ Checking int original:

$$|2-5| \stackrel{?}{=} |2(2)+6|-1$$

$$| -3 | \stackrel{?}{=} | 10 | - 1$$

$$3 \neq 9$$

Abs Value Inequalities

The main idea is that if

$$|f(x)| \leq c \text{ then } -c < f(x) < c$$

and if $|f(x)| > c$ then

$$f(x) < -c \text{ or } f(x) > c$$

But there are other instances such as ...
(to be continued)