

Section 5 -> interest rates, e, compound interest

Definition:

- interest rate -> annual rate at which some money investment or loan without compounding is calculated (simple interest)

- "r"

• say $r = 2\%$

• say $r = .16\% = .0016$

- principal (P) -> initial deposit or loan

- interest -> principal * rate -> $I = Pr$

ex: $P = 200$ $r = .4\%$ annual

$I_{int} = 200 \times .004 = .8$

↳ interest

Now suppose interest is compounded quarterly, that is, every 3 months, $P \cdot \frac{r}{4} =$ interest taken ("distributed")

Leaving that interest in bank _____ the next time the interest is compounded, $[P + (P) (\frac{r}{4})] (\frac{r}{4}) =$ interest

Now you have $[P + (P) (\frac{r}{4})] + [P + (P) (\frac{r}{4})] (\frac{r}{4})$
 ① after 3 months, new principal ② interest after next 3 months

But this is $\underbrace{[P + P \cdot \frac{r}{4}]}_{\text{reduced}} [1 + \frac{r}{4}] \neq \underbrace{P}_{\text{factored}} [1 + \frac{r}{4}] [1 + \frac{r}{4}]$
 $= P [1 + \frac{r}{4}]^2$ new principal after 6 months
 ③

10/3/18

Compound / quarterly after one year:

$$P \left[1 + \frac{r}{4} \right]^4 \text{ account value after 1 year}$$

* Definition:

If P is compounded n times each year then the final account value after 1 year is given by:

$$F = P \left[1 + \frac{r}{n} \right]^n$$

Examples in book - solving for F ; solving for P ; solving for t . But, where's t in formula?

Since n = number of times we compound in a year, then after one year, $n = n \cdot 1$ interest periods after 1 1/2 years, we've got 6 compounding periods. Then,
 $F = P \left(1 + \frac{r}{n} \right)^{nt} = P \left(1 + \frac{r}{n} \right)^{nt}$

$$F = P \left(1 + \frac{r}{n} \right)^{nt}$$

n = # of times compounded per year

t = number of years

nt = number of compound periods, total in t years

$$P = \frac{F}{\left(1 + \frac{r}{n} \right)^{nt}} \rightarrow P = F \left(1 + \frac{r}{n} \right)^{-nt}$$

class notes

$$2 < e < 3$$

ex: $r = \frac{1}{2}\% = .005$ compound monthly over $2\frac{1}{2}$ years, principal \$800

Find F , where $P = 800$, $t = 2\frac{1}{2}$ (2.5), $n = 12$
because monthly

$$F = 800 \left(1 + \frac{.005}{12}\right)^{12(2.5)}$$

Suppose $n = 365$ (ie compounding daily):

$$F = P \left(1 + \frac{r}{365}\right)^{365 \cdot t}$$

As the number of compounding periods $n \rightarrow \infty$, we use that $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$

From this, we get the formula for the account value when P is compounded "continuously"
 \hookrightarrow at every instant

$$F = Pe^{rt}$$

* class notes

- 1 compound interest
- simple interest = double money
- compound continuous interest = almost tripling

10/5/18 Section 5 (foreshadows section 25)

* $F = P(1 + \frac{r}{n})^{nt}$ → number of compound periods total

$n = \text{number of } \frac{\text{compounds}}{\text{year}}$

$t = \text{years}$

final value = F
of account

P = principal investment or loan

Ex 1: $n = \text{semiannual} = 2$

$r = \frac{1}{2}\% = .005$

$\frac{.5}{100}$

18 months = 1.5 years

$$F = P(1 + \frac{.005}{2})^{2(1.5)}$$

$$= P(1 + .0025)^3$$

P = 800

final solution = $800(1.0025)^3$

$n \rightarrow \infty \quad (1 + \frac{r}{n})^{nt} \rightarrow e^{rt}$

So, compounding continuously gives $F = Pe^{rt}$

P = 250 $r = 2\%$ $t = \frac{1}{2}$ year

$n = 1$

$F = 250(1 + \frac{.02}{1})^{1(\frac{1}{2})}$ (16 months)

$= 250(1.02)^{.5}$

↳ on test = answer

Suppose $r = 1 \rightarrow 100\%$ and $n = \infty$ then

$F = Pe^{1 \cdot t}$

$F = Pe^1$

$P \times 2.718$

let $t = 1$ year

For any other n , if $r = 1$

$F = P(1 + \frac{1}{n})^n$

let $n = 1$, i.e. annual simple interest

$F = P(1+1)^1 = P \cdot 2$

Suppose same data, but find how long to
double \$10,000 at 5% (md/mnthly). 10/5/18

$$20,000 = 10,000 \left(1 + \frac{.05}{12}\right)^{12t}$$

$$2 = (1.0042)^{12t}$$

$$\log_2 2 = \log_2 (1.0042)^{12t}$$

$$1 = 12t \log_2 (1.0042)$$

$$1 = 12t \log_2 (1.0042)$$

$$t = \frac{1}{12 \log_2 (1.0042)}$$

$$F = Pe^{rt} \rightarrow \text{to find } t = \frac{\ln 2}{r}$$

↳ continuous compounding

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n finite $F = P(1 + \frac{r}{n})^{nt}$ solve for P $\rightarrow P = F(1 + \frac{r}{n})^{-nt}$

n infinite $F = Pe^{rt}$ $\rightarrow P = Fe^{-rt}$

solve for t

When we want to double P at $r = 1.7\%$ $n = 4$

$$F = P(1 + \frac{r}{n})^{nt}$$

$$2P = P(1 + \frac{.017}{4})^{4t}$$

$$\frac{2P}{P} = \frac{P(1 + \frac{.017}{4})^{4t}}{P}$$

$$2 = (1.00425)^{4t}$$

$$4 \begin{array}{r} .00425 \\ .017000 \\ \hline -16 \\ \hline -10 \\ \hline 20 \\ \hline 20 \\ \hline 0 \end{array}$$

$$\log_2 2 = 4 \log_2 (1.00425)$$

$$1 = \frac{4t \log_2 (1.00425)}{4 \log_2 (1.00425)}$$

$$t = \frac{1}{4 \log_2 (1.00425)}$$

$$F = Pe^{rt}$$

$$2P = Pe^{.017t}$$

$$\ln 2 = (e^{.017t}) \ln 2$$

$$\frac{\ln 2}{.017} = \frac{.017t}{.017} \cdot 1$$

$$t = \frac{\ln 2}{.017}$$

Effective Interest Rate

$$e^r - 1 = e^{.017} - 1$$

$$F = (1 + \frac{r}{n})^n - 1$$

$$F = (1 + \frac{.017}{4})^4 - 1$$

in class notes

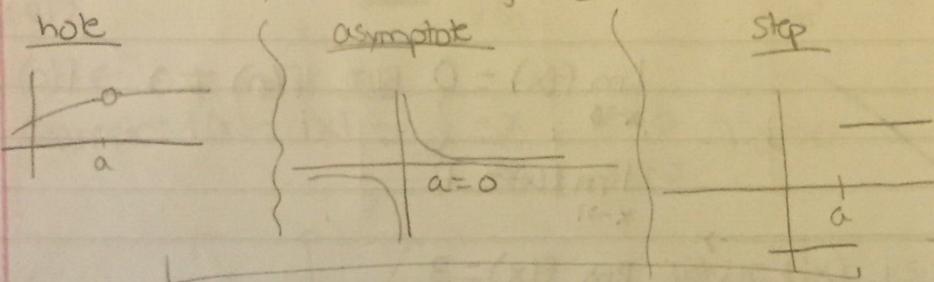
Section 9: Continuity

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- a function that is continuous at some value of $x = a$ does not break at a (pen doesn't lift off paper)

• no hole, no asymptote, no step at $x = a$

↳ where they have these things = NOT continuous



not continuous = discontinuous

review of limits

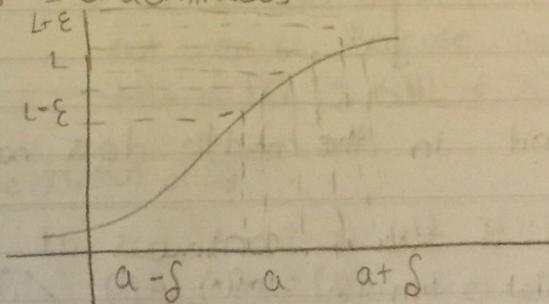
$\lim_{x \rightarrow a} f(x)$ exists (call it L)

if, for any $\epsilon > 0$ there

exists a $\delta > 0$ such that

$$|x - a| < \delta \rightarrow |f(x) - L| < \epsilon$$

arbitrary ϵ but always very small

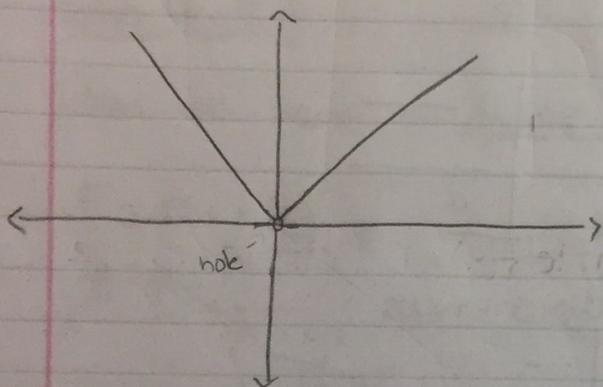


- Functions that have a hole at $x = a$ still have a limit at $x = a$
- Functions that have an asymptote at $x = a$ do not have a limit at $x = a$
- Functions that have a step at $x = a$ do not have a limit at $x = a$

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Piecewise Functions

① $f(x) = \begin{cases} -x, & x < 0 \\ x, & x > 0 \end{cases}$



$\lim_{x \rightarrow 0} f(x) = 0$ BUT, $f(0) \neq 0$ $\rightarrow f(0)$ is not defined $\rightarrow f(0) \neq L$

$\lim_{x \rightarrow 1} f(x) = 1$

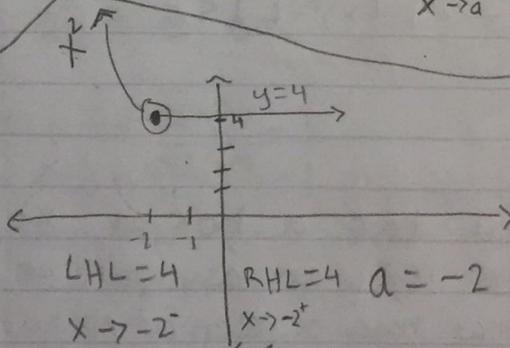
$\lim_{x \rightarrow -3} f(x) = 3$

on test * limit in the middle does not exist \rightarrow It's not enough to have a limit at $x=a$; the value of the limit must be the value of the function at a !

Def: if $f(x)$ is continuous at $a \in \text{Dom } f$ if $\lim_{x \rightarrow a} f(x) = f(a)$
 \hookrightarrow That is $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

② $f(x) = \begin{cases} x^2, & x < -2 \\ 4, & x \geq -2 \end{cases}$

$\underbrace{\hspace{10em}}_y$ $\underbrace{\hspace{10em}}_x$
 (domain)



$RHL = LHL \rightarrow$ limit exists

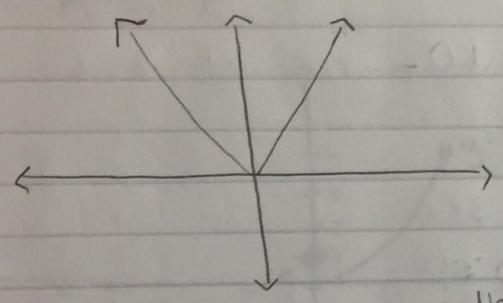
Piecewise Functions \rightarrow check all interval endpoints
 \hookrightarrow check $f(a)$ if a is at end of an interval of domain

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2 things that have to hold true

a function $f(x)$ is differentiable at $x=a$ if it is continuous there and $\lim_{x \rightarrow a^-} \text{DQ} = \lim_{x \rightarrow a^+} \text{DQ}$

example: $f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases} \quad a=0$



while $f(x)$ is continuous at $x=a$ the derivative, $f'(x)$ does not exist. Because the $\text{LHL of DQ} \neq \text{RHL of DQ}$

alternative definition of DQ

DQ limit

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

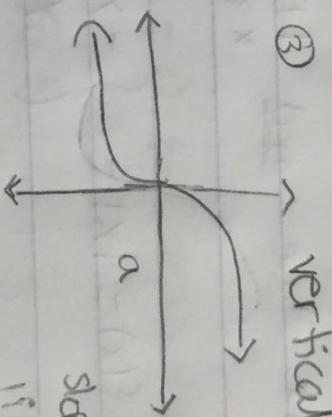
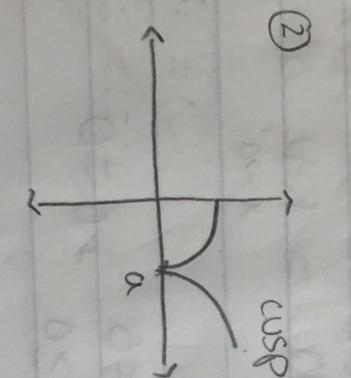
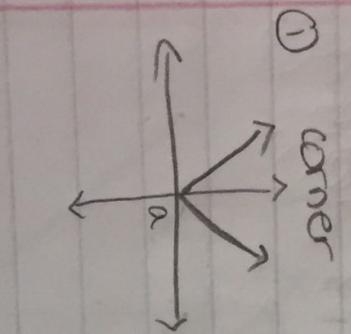
LHL $\lim_{x \rightarrow 0^-} \frac{f(x) - f(a)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x - 0}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = \boxed{-1}$

RHL $\lim_{x \rightarrow 0^+} \frac{x - 0}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x}{x} = \boxed{1}$

not equal
The LHL \neq RHL
at $x=0$, so
 $f(x)$ is not
differentiable at
 $x=0$

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3 examples where $f(x)$ is continuous at $x=a$ but not differentiable (has no derivative so $f'(x)$ DNE)



vertical tangent
slope of vertical line = undefined (∞)

test \rightarrow chapters 15-21 (omit 16)
 \hookrightarrow chapter 20 = extra practice

★ on test

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criteria for continuity at $x=a$

- 1.) $f(a)$ exists
- 2.) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$
- ✓ 3.) $f(a) = L$

on test

Ex. Show why $x=2$ is a point of discontinuity in the graph

$$f(x) = \begin{cases} x-1 & x < 2 \\ 0 & x = 2 \\ 1 & x > 2 \end{cases}$$

steps

① $f(a)$ exists $\rightarrow f(2) = 0$

② $\lim_{x \rightarrow 2^+} f(x) = 1$

$\lim_{x \rightarrow 2^-} f(x) = 1$

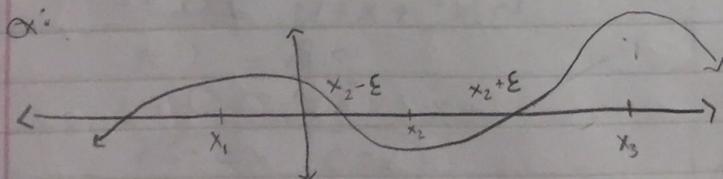
③ $0 \neq 1$

Textbook:

- \rightarrow chapter 19 \rightarrow graphing polynomials (no asymptotes hor v)
- \rightarrow chapter 20 \rightarrow rational functions, exponential functions, log functions
 - have asymptotes \rightarrow vertical + horizontal
- \rightarrow chapter 21 \rightarrow curve sketching for sections 19 and 20
- \rightarrow chapters 15, 17, 18 \rightarrow extremes of a function
 - \hookrightarrow highest and lowest

• firstly \rightarrow talk about local extremes

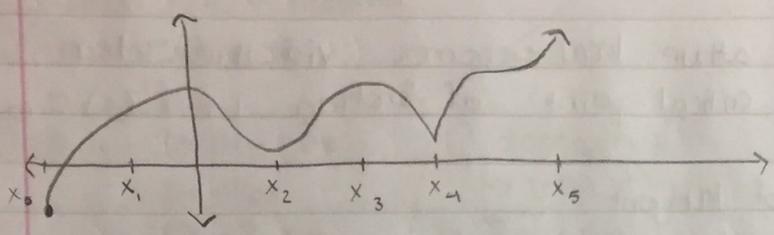
ϵ neighborhood of x_1



* " ϵ " arbitrarily small interval value

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ex:

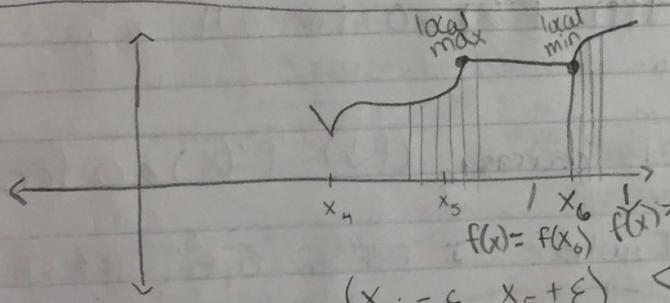


* Definition: f has a local maximum at $x=a$ if there is $\epsilon > 0$ such that for any x in an ϵ -neighborhood of a , $f(x) \leq f(a)$

arbitrarily small integer value

- For all $x \in (a-\epsilon, a+\epsilon)$ $f(x) \leq f(a)$. So $f(a)$ is a local max

→ needed to be because if something is a local max does not mean that there isn't another local max



$(x_5 - \epsilon, x_5 + \epsilon)$
 left of x_5 $f(x) < f(x_5)$
 right of x_5 $f(x) \geq f(x_5)$

* definition: f has a local minimum at $x=a$ if there is $\epsilon > 0$ such that for any x in an ϵ -neighborhood of a , $f(x) \geq f(a)$

- slope of vertical
line = undefined

- all local extremes = critical points

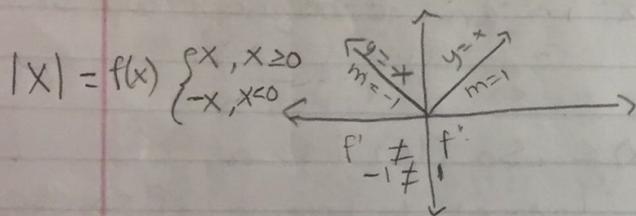
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Tasks: find where $f(x)$ attains local extremes via the idea that c is a critical point of function if $f'(c) = 0$ or $f'(c) = \text{DNE}$

- value of derivative = slope of tangent

* - can't draw tangents unless the curve is smooth

* $f(x) = |x|$ has no derivative at $x=0$



- At all local "extremes", $f'(x) = 0 \dots$ on intervals where f is increasing ($f \uparrow$) $f'(x) > 0$ for all x in interval

- On intervals where f is decreasing ($f \downarrow$) $f'(x) < 0$ for all $x \in I$

- shorthand $(x - \epsilon, x + \epsilon) = I$, where I is an open interval of x

$$f(x) = x^{1/3}$$
$$f'(x) = \frac{1}{3} x^{-2/3}$$
$$0 = \frac{1}{3x^{2/3}}$$

DNE at $x=0$ (set denominator $3x^{2/3} \neq 0$)
because $3x^{2/3} = 0$ at $x=0$ (can't have a 0 downstairs)

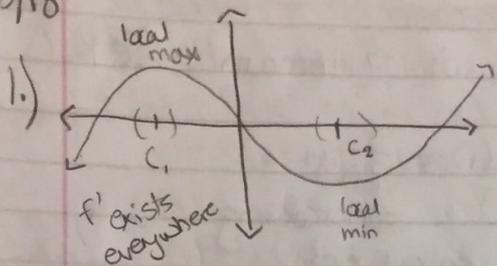
*class notes

$\epsilon \rightarrow$ some arbitrarily small distance

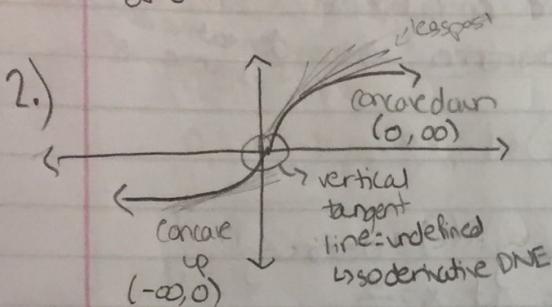
*can have negative local max ~~the~~ f_{max}

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*must give intervals of \uparrow, \downarrow of concave up and down



polynomial function
max $\rightarrow f(x) \leq f(c_1)$
min $\rightarrow f(x) \geq f(c_2)$



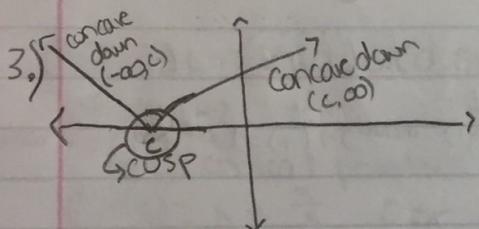
root function

$f(x) = x^{1/n}$ (n is odd)
no extrema

Slope more positive means the derivative is positive

$f' \uparrow$ then $f'' \uparrow$ (greater than)

$f' \downarrow$ then $f'' \downarrow$ (less than)

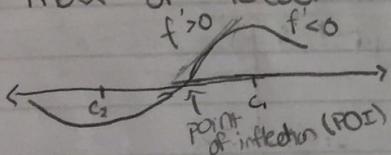


root function squared

$f'(c) \rightarrow$ DNE
 $f'(b); f''(b) =$ DNE

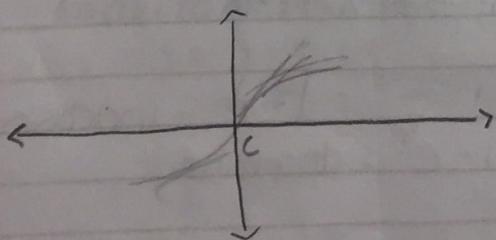
*value of extrema is $f(x)$

First Derivative tests for whether the critical value is a local max or local min



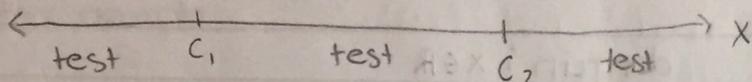
$f(c_1)$ is local max if $f' > 0, x \leq c_1$
 $f' < 0, x \geq c_1$

$f(c_2)$ is a local min if $f' < 0, x \leq c_2$
 $f' > 0, x \geq c_2$



$f'(c)$ DNE, c critical point
but $f'(x) > 0$ for all x
except $x=c$

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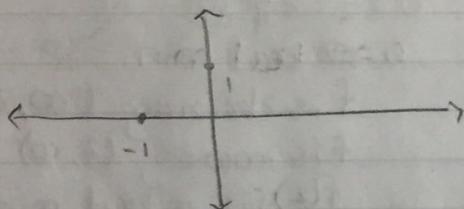


(do a sign analysis of the derivative)

Question 2h.) on HW section 17

$$a^{4/5} = (a^4)^{1/5} \text{ or } (a^{1/5})^4$$

- any odd root function $y = x^{1/n}$ has this shape (below)
 $y = 1 + x^{1/5}$ is just moved up one unit



look at HW PDF

not on test

- * $s(t)$ displacement
- $s'(t) = v(t)$ velocity
- $s''(t) = v'(t) = a(t)$ acceleration

- rate of the rate of change of the original function

to find where concavity changes

Second Derivative Test \rightarrow tests for where concavity changes from up to down or vice versa

point of inflection \rightarrow where concavity changes
 \hookrightarrow generally where $f'' = 0$

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odd root
function
is odd
real #

in class example

domain: $x \in \mathbb{R}$

$$f(x) = (x-2)^{2/3} + 1$$

$$f'(x) = \frac{2}{3}(x-2)^{-1/3} \cdot (1) \rightarrow \frac{2}{3(x-2)^{1/3}}$$

$$f''(x) = -\frac{2}{9}(x-2)^{-4/3} \cdot 1$$

$$0 = \frac{2}{3}(x-2)^{-1/3} \cdot 1$$

$$0 = \frac{2}{3}(x-2)^{-1/3}$$

$$0 = (x-2)^{-1/3}$$

$$0 = x-2$$

$2 = x$ critical point

$$f(2) \rightarrow \text{DNE}$$

FDT decreasing increasing

$$f'(1) = \frac{2}{3(1-2)^{1/3}}$$

$$= \frac{2}{3(-1)^{1/3}}$$

= local min

f is decreasing $(-\infty, 2)$

f is increasing $(2, \infty)$

$f(2)$ is a local min

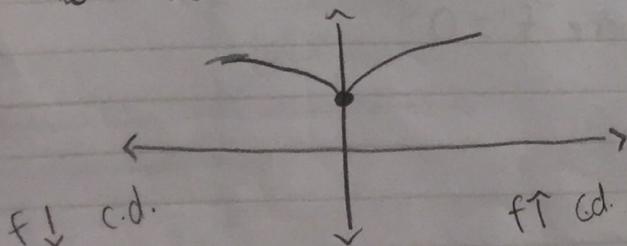
$$f''(x) = \frac{-2}{9(x-2)^{4/3}}$$

SDT

$$f(1) = \frac{-2}{9(1-2)^{4/3}} = \frac{-}{+} = - \text{ (decreasing) } \rightarrow \text{concave down}$$

$$f(3) = \frac{-2}{9(3-2)^{4/3}} = \frac{-}{+} = - \text{ decreasing } \rightarrow \text{concave down}$$

*graph has a cusp (local min and concave down on both sides)



another example

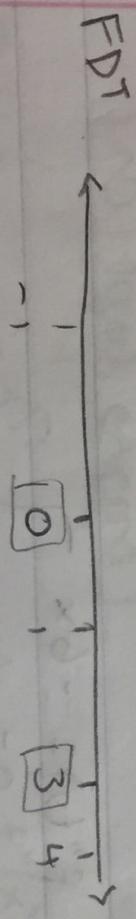
$$f(x) = x^4 - 4x^3 + 10$$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0$$

$x=0, x=3$ critical values \rightarrow FDT

$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

$x=0, x=2$ critical values \rightarrow SDT

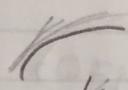


$$f'(0) = 4(0)^3 - 12(0)^2 = 0$$

$$f'(3) = 4(3)^3 - 12(3)^2 =$$

-class notes

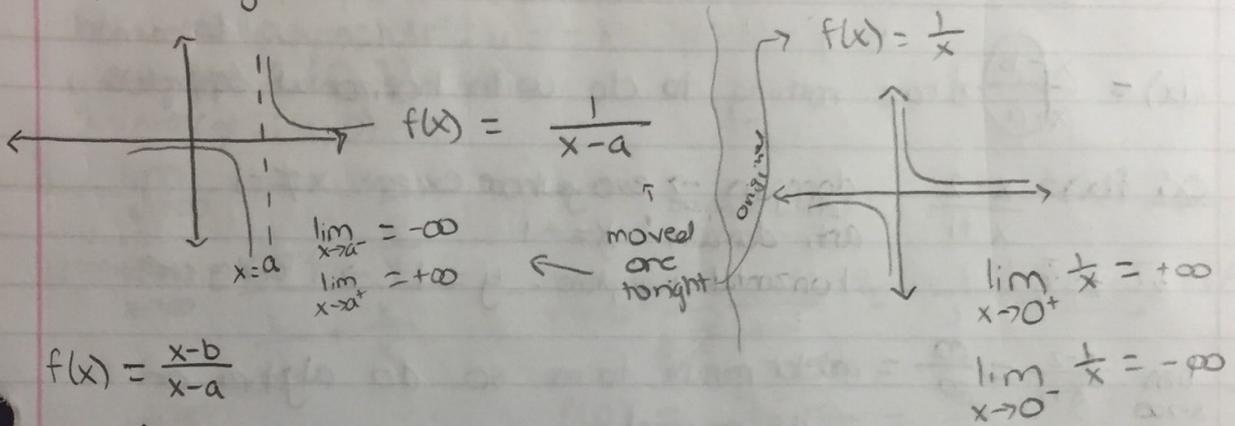
on test

* when $f' \downarrow$, so $f'' < 0$ (negative)  gets flatter
 * when $f' \uparrow$, so $f'' > 0$ (positive)  gets steeper

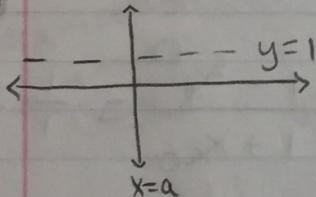
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Section 21

-steps on page \rightarrow read



$f(x) = \frac{x-b}{x-a}$



-vertical axis = anti-domain

• what isn't in the domain is the vertical asymptote

-horizontal asymptote

• the ratio of the leading coefficients when the degree of the numerator and denominator are equal

ex: $\frac{x-b}{x-a} = \frac{1}{1} = \boxed{1} \rightarrow$ horizontal asymptote

new information

-horizontal asymptotes (HA)

• the value of the function as x goes to negative and positive infinity

• to find the HA \rightarrow determine the limit of the function as $x \rightarrow \infty$ and $x \rightarrow -\infty$

$\lim_{x \rightarrow \infty} f(x) = L$ HA: $y = L$
 $\lim_{x \rightarrow -\infty} f(x) = L$

$\frac{\infty}{\infty}$ = indeterminate form so do algebra on original

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Horizontal Asymptote (y=0)

$\lim_{x \rightarrow \infty} \frac{1}{x} = 0$

$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0$

$f(x) = \frac{x-b}{x+a}$ have nothing to do with horizontal asymptote

Ex: $f(x) = \frac{x-2}{x+1}$

domain \rightarrow everywhere except $x \neq -1$

vertical asymptote
anti-domain: $x = -1$

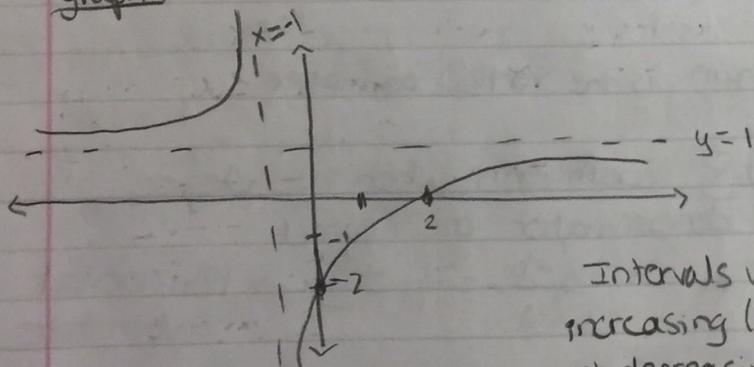
horizontal asymptote: $y = 1 \rightarrow \div$ leading coefficients

* $\lim_{x \rightarrow \infty} \frac{x-2}{x+1} = \frac{\infty}{\infty}$ = indeterminate form so do algebra on the original

$\lim_{x \rightarrow -\infty}$

$\lim_{x \rightarrow \infty} \frac{\frac{x}{x} - \frac{2}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{2}{x}}{1 + \frac{1}{x}} = \frac{1}{1} = 1$

graph:



$f(0) = \frac{0-2}{0+1} = -2$

$f(x) = \frac{x-2}{x+1} = 0$ at $x=2$
only care about numerator

Intervals where:
increasing $(-\infty, -1) \cup (-1, \infty)$
not decreasing anywhere
concave up/down
 $(-\infty, -1) \rightarrow (-1, \infty)$

POI: NO POI because
an asymptote is not in
the domain ($x = -1$ not in
the domain)

$\frac{x-1}{x^2}$

*leading coefficient \rightarrow highest degree

$$\begin{array}{r} 4-x=0 \\ -4 \\ \hline -x = -4 \\ \hline x = 4 \end{array}$$

$$\frac{A}{B} = 0 \Rightarrow A = 0$$

ex: $f(x) = \frac{4-x}{2+x}$

domain: everywhere except $x \neq -2 \rightarrow (-\infty, -2) \cup (-2, \infty)$

vertical asymptote: $x = -2$

horizontal asymptote: $y = -1$

y-intercept: $(0, 2)$

x-intercept: $(4, 0)$

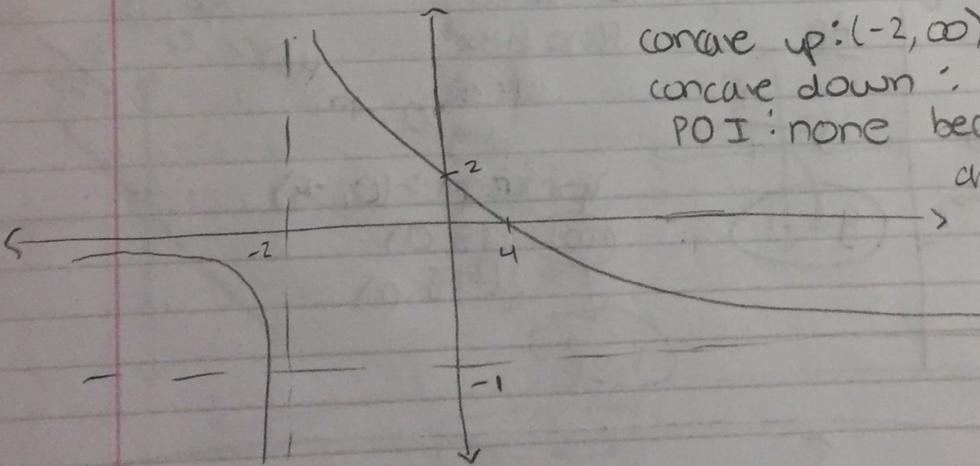
$\lim_{x \rightarrow \infty}$

$$\lim_{x \rightarrow \infty} \frac{\frac{4}{x} - \frac{x}{x}}{\frac{2}{x} + \frac{x}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{4}{x} - 1}{0 + 1} = \frac{-1}{1} = \boxed{-1} \text{ HA}$$

$$f(0) = \frac{4-0}{2+0} = \frac{4}{2} = \boxed{2} \leftarrow \text{y-intercept}$$

*x-intercept
 $4-x=0$
 $x=4$

graph



concave up: $(-2, \infty)$

concave down: $(-\infty, -2)$

POI: none because even though it changes concavity $x = -2$ is not in the domain because $x = -2$ is an asymptote

$$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow \infty} \frac{ax^n + 1}{bx^{n-1}} = \frac{a}{b} = \text{ratio}$$

$$\lim_{x \rightarrow \infty} \frac{ax^n + 1}{x-1} = \infty$$

* denominator degree is bigger \rightarrow limit = 0

* degree same take ratio of leading coefficient

* numerator degree higher \rightarrow limit = ∞

ex: vertical asymptote + horizontal asymptote

① VA

$$7x^2 - 1 = 0$$

$$7x^2 = 1$$

$$\sqrt{x} = \sqrt{\frac{1}{7}}$$

$$x = \left(\frac{1}{7}\right)^{1/2}$$

$$x = \pm \sqrt{\frac{1}{7}}$$

~~HA~~ $y = \frac{1}{7}$

y-intercept \rightarrow

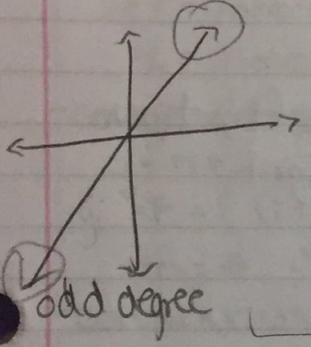
$$\frac{x^2 - 2x + 3}{7x^2 - 1} = \text{use quadratic formula}$$

10/17/18

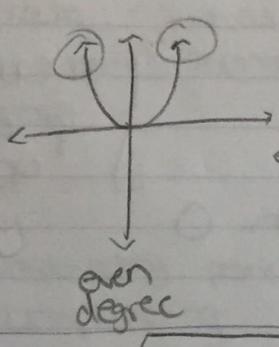
Precalculus facts about polynomials (end behavior)

use power functions $y=x^n$ as parent functions to investigate end behavior of $y=a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ where a_n is your leading coefficient and n is degree

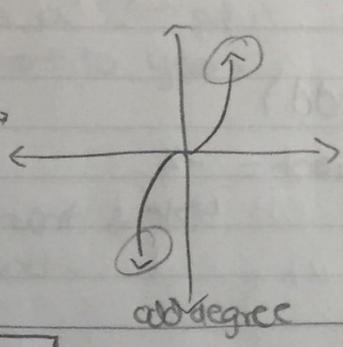
$y=x$



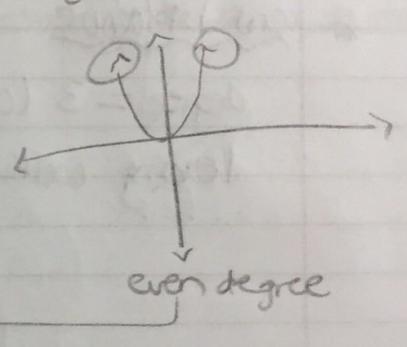
$y=x^2$



$y=x^3$

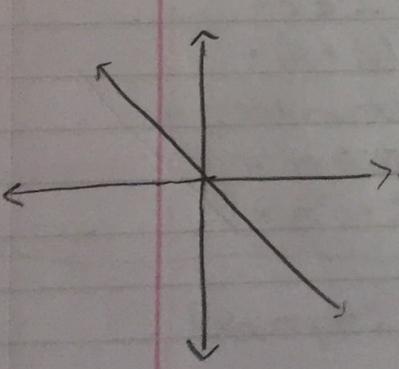


$y=x^4$

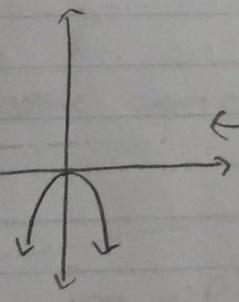


$a_n > 0$

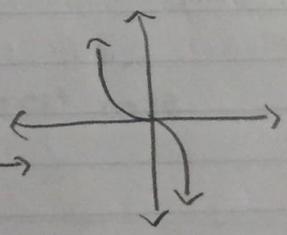
$y=-x$



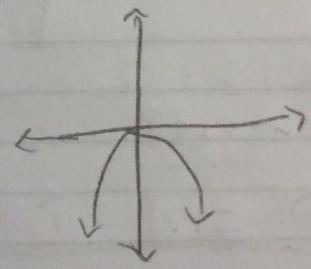
$y=-x^2$



$y=-x^3$



$y=-x^4$



$a_n < 0$

10/17/18

Ex $f(x) = -2x^3 + 3x^2 - 4x + 1$
FDT $\rightarrow f'(x) = -6x^2 + 6x - 4 \rightarrow -2(3x^2 - 3x + 2) = 0$ \rightarrow find critical points
SDT $\rightarrow f''(x) =$

* plug into ^{critical} second derivative $\rightarrow f''(c) > 0$ then local min
 $f''(c) < 0$ then local max

* end behavior
degree = 3 (odd)
leading coefficient = -2
 \rightarrow less than 0

} means
up on left, down
on right

-variable in exponent \rightarrow take \ln on both sides

Tutoring notes interest \rightarrow chapter 5

10/18/18

- Simple interest

- $n = 1$

- $F = P(1+r)^t$

- $n = \#$ of compounding periods

- $F =$ future value \rightarrow how much \$ you have in the end

- $P =$ principal value \rightarrow initial investment

- $r =$ interest rate \rightarrow given as a % \rightarrow put it in the formula as a decimal

- $t =$ time (in years) \rightarrow convert to years

- compound interest

- $n > 1$

- $F = P \left(1 + \frac{r}{n}\right)^{nt}$

- $n = \#$ of compounding periods

- continuous compounding

- $n = \infty$

- $F = Pe^{rt}$

- effective interest rate

- $n > 1$

- $\left(1 + \frac{r}{n}\right)^n - 1$

- continuous effective interest rate

- $n = \infty$

- $e^r - 1$

Example: $\frac{200}{100} = \frac{100 e^{.05t}}{100}$

$$\ln(2) = (e^{.05t}) \ln$$

$$\frac{\ln 2}{.05} = \frac{.05t}{.05}$$

$$t = \frac{\ln 2}{.05}$$

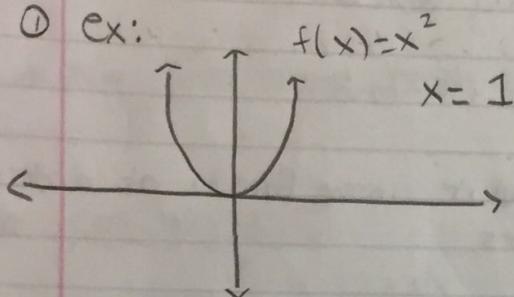
10/18/18

* value of function at a point must equal value of limit at that point and that means that the limit is continuous

Continuity

- draw a graph without lifting your pen = continuous
- function is continuous at a spot if $f(a) = \lim_{x \rightarrow a} f(x)$

① ex:

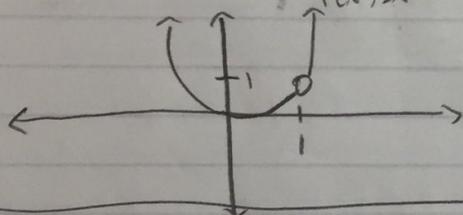


$$f(1) = 1^2 = 1$$

$$\lim_{x \rightarrow 1} x^2 = 1$$

} Same = continuous
when not same = discontinuous

② ex: $f(x) = \begin{cases} x^2, & \text{all } x \neq 1 \\ -74, & x = 1 \end{cases}$



$$\lim_{x \rightarrow 1} x^2 = 1^2 = 1$$

$$f(1) = -74$$

* important!: If f is not continuous at $x = a$ then $f'(a) = \text{DNE}$

FDT and SDT

$$f(x) = \frac{1}{3}x^3 + 3x^2 - 40x + 74$$

$$f'(x) = x^2 + 6x - 40$$

$$f''(x) = 2x + 6$$

- function gives you the values at certain points
- first derivative tells you if function is increasing or decreasing on a certain interval
- second derivative tells you if function is concave up or concave down, or **POI** (where concavity changes)

10/18/18

- you can plug critical points from first derivative into second derivative to determine local max or local min

↳ ex: $f(x) = x^3 - 4x^2$

$$f'(x) = 3x^2 - 8x = 0 \rightarrow x(3x - 8) = x = 0, x = \frac{8}{3}$$

$$f''(x) = 6x - 8$$

$$f''(0) = 6(0) - 8 = -8 \text{ (at } 0 \text{ negative concavity = max)}$$

$$f''\left(\frac{8}{3}\right) = 6\left(\frac{8}{3}\right) - 8$$

$$= \frac{48}{3} - 8$$

$$= 16 - 8$$

$$= 8 \text{ (at } \frac{8}{3} \text{ positive concavity = min)}$$

- to find critical point $(x, y) \rightarrow$ plug critical point into the original function

test review → in class

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

→ continuous compounding interest

$$t = \frac{\ln 2}{r}$$

10/19/18

Find time (in years) for an investment of \$750 to double when interest at $1\frac{3}{4}\%$ is compounded:

1.) monthly

$$\frac{1500}{750} = \frac{750 \left(1 + \frac{.0175}{12}\right)^{12t}}{750}$$

$$2 = \left(1 + \frac{.0175}{12}\right)^{12t}$$

$$\log_2 2 = \log_2 \left(1 + \frac{.0175}{12}\right)^{12t}$$

$$\frac{1}{12 \log_2 \left(1 + \frac{.0175}{12}\right)} = \frac{12t \log_2 \left(1 + \frac{.0175}{12}\right)}{12 \log_2 \left(1 + \frac{.0175}{12}\right)} \rightarrow t = \frac{1}{12 \log_2 \left(1 + \frac{.0175}{12}\right)}$$

$$\begin{array}{r} 1.4 \ 0014583 \\ 12 \overline{) .01750} \\ \underline{- 12} \\ 55 \\ \underline{- 48} \\ 70 \\ \underline{- 60} \\ 100 \\ \underline{- 96} \\ 40 \\ \underline{- 36} \end{array}$$

2.) continuously

$$F = Pe^{rt}$$

$$\frac{1500}{750} = \frac{750 e^{.0175t}}{750}$$

$$\ln(2) = \left(e^{.0175t}\right) \ln$$

$$\frac{\ln 2}{.0175} = \frac{.0175t}{.0175}$$

$$\frac{\ln 2}{.0175} = t$$

3.) effective interest rate

$$e^{.0175} =$$

Definition of f is continuous at $x=a$

* ① $a \in \text{Domain of } f \rightarrow f(a)$ is a value of the function $\rightarrow f(a)$ is defined

② graph \rightarrow can't lift pen from paper

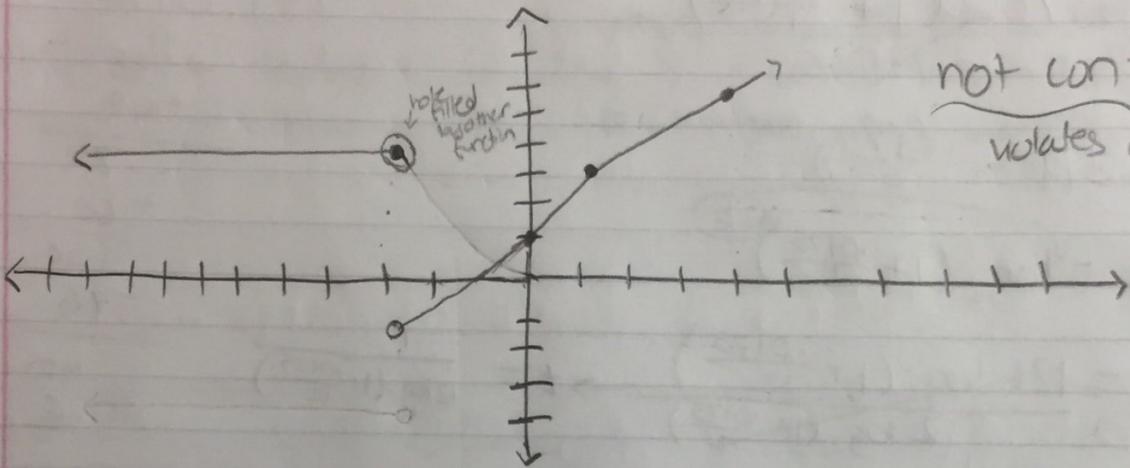
③ $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L \rightarrow \text{RHL} = \text{LHL} \rightarrow \text{limit exists at } x=a$

④ $L = f(a)$
value of limit has to equal $f(a)$

-in class

10/11/18 test review

ex:1 $f(x) = \begin{cases} 4 & x < -2 & \rightarrow (-2, 4) \\ x^2, & x = -2 & \rightarrow -2^2 = 4 \rightarrow (-2, 4) \\ x+1, & x > -2 & \rightarrow -2+1 = -1 \rightarrow (-2, -1) \end{cases}$



not continuous
violates criteria

ex:2 $f(x) = \begin{cases} 4 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 2 \\ x^2 - 2 & \text{if } x > 2 \end{cases} \left. \begin{array}{l} a=2, a=0 \\ \rightarrow \text{RHL} = \text{LHL} \end{array} \right\}$

