

Steps for Solving "Find equation of the line tangent to _____" original equation

Step 1: Use $y - f(x_1) = f'(x_1)(x - x_1)$ as your outline/guide

Step 2: make a list based on what you have and what you need

Step 3: Find what is missing:

1) if you need x , use the given y value and plug it into the original equation. If you need y , use the given x value and plug it into the original equation.

2) when you need to find $f'(x_1)$, use the derivative shortcuts and rules to find the derivative of the original equation. Once you have the derivative, plug your x value into the derivative and solve.

Step 4: Plug your pieces into the outline and simplify

Example

Find the equation of the line that is tangent to $f(x) = 2\sqrt{x} - 3x$, at $x_1 = 1$

Step 1: $y - f(x_1) = f'(x_1)(x - x_1)$

Step 2: $x_1 = 1$, $f(x_1) = ?$ $f'(x_1) = ?$

Step 3: Finding $f(x_1)$:

$$f(x) = 2\sqrt{x} - 3x \rightarrow f(x) = 2\sqrt{1} - 3(1) = f(x) = 2 - 3 = f(x_1) = -1$$

Finding $f'(x_1)$:

$$f(x) = 2\sqrt{x} - 3x \rightarrow f'(x) = 2x^{1/2} - 3x \rightarrow f'(x) = \frac{1}{2} \cdot 2x^{1/2-1} - 3$$

$$f'(x) = (1)^{-1/2} - 3 \rightarrow f'(x) = \frac{1}{\sqrt{x}} - 3, \text{ now plug } x_1 \text{ into } f'(x)$$

$$f'(x) = \frac{1}{\sqrt{1}} - 3 \rightarrow f'(x) = -2$$

Step 4: $y - f(x_1) = f'(x_1)(x - x_1)$

$$y - (-1) = -2(x - 1) \rightarrow y + 1 = -2x + 2$$

$$\boxed{y = -2x + 1}$$

RulesProduct Rule

$$[f(x) \cdot g(x)]' = f'(x)g(x) + g'(x)f(x)$$

example

$$y = xe^x \quad f = x \quad g = e^x$$

$$y' = (1 \cdot e^x) + (e^x \cdot x)$$

$$y' = e^x(1+x)$$

Quotient Rule

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{f'g - g'f}{g^2}$$

example

$$y = \frac{e^x}{8} \quad f = e^x \quad g = 8$$

$$y' = \frac{e^x \cdot 8 - 0 \cdot e^x}{8^2} \quad y' = \frac{8e^x}{8^2} = \boxed{\frac{e^x}{8}}$$

Reciprocal (baby quotient) Rule

$$y = \frac{c}{f(x)} \quad y' = \frac{-c}{(f(x))^2} \cdot f'(x)$$

or via quotient rule: $\frac{0 \cdot f(x) - f'(x) \cdot c}{(f(x))^2}$

example $y = \frac{5}{\sqrt{x}} \quad c = 5 \quad f(x) = \sqrt{x}$

$$y' = \frac{0 \cdot \sqrt{x} - \frac{1}{2\sqrt{x}} \cdot 5}{(\sqrt{x})^2} = \frac{-\frac{1}{2\sqrt{x}} \cdot 5}{x}$$

$$y' = \frac{-5}{2\sqrt{x} \cdot x} = \boxed{y' = \frac{-5}{2x\sqrt{x}} \text{ or } \frac{-5}{2x^{3/2}}}$$

Chain Rule

$$y = n [u(x)]^{n-1} \cdot u'(x)$$

★ note: $u'(x)$ is the same as $\frac{du}{dx}$, which represent the derivative of $u'(x)$

example

$$y = (x+2)^9, \text{ what is } y'?$$

$$y' = 9[x+2]^{9-1} \cdot 1 \rightarrow \boxed{y' = 9(x+2)^8}$$

simple

U Forms

advanced

Power Rule

$$1) y = x^n \rightarrow \frac{dy}{dx} = n \cdot x^{n-1}$$

example: $y = x^7$

$$\frac{dy}{dx} = 7 \cdot x^{7-1} = 7x^6$$

expon-2) rule

$$y = e^x, \frac{dy}{dx} = e^x$$

$$3) y = \ln x, \frac{dy}{dx} = \frac{1}{x}$$

$$y = [u(x)]^n \rightarrow \frac{dy}{dx} = n \cdot u^{n-1} \cdot u'(x)$$

example: $y = [4x-1]^7$

$$\frac{dy}{dx} = 7(4x-1)^{7-1} \cdot 4 = 7(4x-1)^6 \cdot 4$$

$$y = e^{u(x)}, \frac{dy}{dx} = e^{u(x)} \cdot \frac{du}{dx}$$

example: $y = 5e^{3x}$

$$\frac{dy}{dx} = 5e^{3x} \cdot 3$$

$$y = \ln(u(x)), \frac{dy}{dx} = \frac{1}{u(x)} \cdot \frac{du}{dx}$$

example: $y = \ln(5x+1)$

$$\frac{dy}{dx} = \frac{1}{5x+1} \cdot 5$$

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Practice Problems

$$f(x) = 5x(2 - \ln x)^3$$

$$f = 5x \quad g = (2 - \ln x)^3$$

The Product Rule

$$f'(x) = 5 \cdot (2 - \ln x)^3 + \left(3[2 - \ln x]^{3-1} \cdot -\frac{1}{x} \right) \cdot 5x$$

this the chain rule. The derivative of $2 - \ln x = 0 - \frac{1}{x}$

$$f'(x) = 5 \cdot (2 - \ln x)^3 + \left(3[2 - \ln x]^{3-1} \cdot -\frac{1}{x} \right) \cdot 5x$$

★ you do not need to simplify this for the exam on 09/27, but you do need to simplify things like $e^0 (=1)$, $\sqrt{1} (=1)$ and $\sqrt{0} (=0)$

$$y = (6 - 2x^4)^3$$

The Chain Rule

$$\frac{dy}{dx} = 3[6 - 2x^4]^{3-1} \cdot \frac{du}{dx}$$

$$\begin{aligned} \frac{du}{dx} &= \frac{6 - 2x^4}{0 - 2 \cdot 4x^{4-1}} \\ &= -2 \cdot 4x^3 \end{aligned}$$

$$\frac{dy}{dx} = 3[6 - 2x^4]^{3-1} \cdot -2 \cdot 4x^3$$

$$y = (3 + \sqrt{x})^6$$

The Chain Rule

$$y' = 6[3 + \sqrt{x}]^{6-1} \cdot \frac{du}{dx}$$

$$3 + \sqrt{x}$$

$$\frac{du}{dx} = 0 + \frac{1}{2\sqrt{x}} \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$y' = 6[3 + \sqrt{x}]^5 \cdot \frac{1}{2\sqrt{x}}$$

$$y = (\ln x)^3$$

$$y' = 3[\ln x]^{3-1} \cdot \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$y' = 3[\ln x]^2 \cdot \frac{1}{x}$$

$$y = .1e^{-5x}$$

Use: $y = e^{kx} = y' = ke^{kx}$
 $y' = -5 \cdot .1e^{-5x}$ $y' = -.5e^{-5x}$

$$y = \ln(cx) = y' = \frac{1}{x}$$

$y = \ln(cx)$ has the same derivative ($y' = \frac{1}{x}$) as $y = \ln(x)$ because $y = \ln(cx) = \ln c + \ln x$
 $\downarrow$ $\downarrow$
0 $+\frac{1}{x}$

$y = \ln u$ the derivative is $y' = \frac{1}{u} \cdot \frac{du}{dx}$

example

$$y = \ln(3x+2) \quad y' = \frac{1}{3x+2} \cdot \frac{du}{dx} \quad \frac{du}{dx} = 3+0$$

$$y' = \frac{1}{3x+2} \cdot 3 = \frac{3}{3x+2} = y'$$

$$y = [\ln(3x+2)]^5 \quad y' = 5[\ln(3x+2)]^{5-1} \cdot \frac{du}{dx} \cdot \frac{dv}{dx}$$

$$y' = 5[\ln(3x+2)]^4 \cdot \frac{1}{3x+2} \cdot 3$$

$$y = (\sqrt{x} \cdot 3^x) + \left(\frac{4x}{\ln x}\right) \rightarrow 4x = f \quad \ln x = g$$

$$f = \sqrt{x} \quad g = 3^x$$

quotient rule: $\frac{f'g - g'f}{g^2} = \frac{4 \cdot \ln x - \frac{1}{x} \cdot 4x}{(\ln x)^2}$

product rule: $f'(g) + g'(f)$

$$\frac{1}{2\sqrt{x}}(3) + 3^x \ln 3(\sqrt{x})$$

$$y' = \left(\frac{1}{2\sqrt{x}} \cdot 3\right) + \left(3^x \cdot \ln 3 \cdot \sqrt{x}\right) + \left(\frac{4 \cdot \ln x - \frac{1}{x} \cdot 4x}{(\ln x)^2}\right)$$

Content for Exam 1

- 1) Essential Functions to be drawn/shown via piecewise function
- 2) Domain (of essential functions and other functions) and express in interval notation
- 3) Equation of a line tangent to $f(x)$ at a point x or y
- 4) marginal analysis (expect word problems) of non-linear examples (see 2.5 handout)
- 5) demand equation: $p(x)$, $p(q)$, $q(x)$ or $q(p)$
- 6) a small amount of limit computation
- 7) interpret marginal cost, revenue, etc.
for ex) $c'(30) = \$12$, means the production of the 31st unit costs about \$12, unless $c(x)$ is a linear function, in which the marginal cost is exactly \$12
- 8) Derivative computation, one including the f' def $\left(\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \right)$ and shortcuts