

Unit Ch 1-4

Ch. 3 Polynomials + Rational Fns.

Vocab Term = monomial

Examples 3, $\frac{x}{2}$, $-7x^2y$ All these entail
only (\times) of variables
and numbers

$\frac{1}{2}x$ you cannot have a
variable in a denominator!

Polynomial - Sum (or difference) of monomials

BTW, a monomial is a single term polynomial

Example $x^2 - 4x + 16$

So far, we have only expressions, that is,
not equations.

Degree of a term (monomial): ~~highest~~ power

Degree of a polynomial: highest power
of the monomials

Example ④ 28 deg zero

① $3x^2 - 6$ deg 2 or second deg

② xy^2z^3 deg $6 (1+2+3)$

③ $4m^2n^1 - mn^2 + 7mn - 1$ deg $3 (2+1)$

Degree of a constant (number) is zero.

Ex -17 deg zero

BTW, $x^0 = 1$, $x \neq 0$ b/c 0^0 is not def'd

Quick primer on exponent rules

base
exponents $m, n \in \mathbb{Z}$ and $ka \in \mathbb{R}$, the following properties apply:

1. $a^0 = 1$, $a \neq 0$

2. $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$

3. $a^n + a^m$ cannot be combined

b/c a^n, a^m are not "like terms"
i.e., variables with the same exponent.

4. $8a^n - 2a^n = 6a^n$

Combine like terms

5. $a^n \cdot a^m = a^{n+m}$

ex $(-6a^4)(12a^{-1}) = -72a^3$

6. $\frac{a^n}{a^m} = a^{n-m}$

7. $(\frac{a}{b})^n = \frac{a^n}{b^n}$

8. $a^{-n} = \frac{1}{a^n} \Rightarrow \frac{1}{a^{-n}} = a^n$

Polynomial expressions are common for describing cost, revenue, profit

Another expression, rational expression, also occurs in applications.

$$\frac{4}{x} + 2x^2 - 60 \quad \text{b/c } \frac{4}{x} = 4x^{-1}$$

is not a monomial
b/c a variable is divided here.

Formal def. of polynomial & rational function

(1) Polynomial function is modeled by this:

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where a_n, a_{n-1}, \dots are coefficients and x^n, x^{n-1}, \dots are the variables and this polynomial fn is named $f(x)$, where $f(x) = y$, a different variable.

"y is a fn of x"

But it could be ~~any~~ other letters,
for ex:

cost ~~q~~ is a fn. of quantity produced

Ex $C(q) = q^2 - 70q + 200$

② Rational fun. $y = f(x) = \frac{P(x)}{Q(x)}$

where $P(x)$ and $Q(x)$ are polynomials
and $Q(x) \neq 0$ for any but perhaps
a few values of x

Ex $y = \frac{4}{x}$ $P(x) = 4$
 $Q(x) = x$

9/3 Important lines: Horizontal, $m=0$, $y=b$

Vertical, m undefined, $x=a$

All others: m is nonzero, $y=mx+b$

Domain: Set of all values of x for which $f(x)$ is defined. What are the main restrictions we're referring to?

I. For $f(x) = \text{polynomial}$, none. $f(x)$ can be evaluated for any real no. We write $x \in \mathbb{R}$ or $(-\infty, \infty)$

II. For rational funs, $f(x) = \frac{P(x)}{Q(x)}$, $Q(x) \neq 0$, so set $Q(x) \neq 0$ and solve for x . The domain is all $x \neq$ these values in \mathbb{R} .

Ex $f(x) = \frac{6-x}{x^2-3}$ $Q(x) \neq 0$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm \sqrt{3}$$

$$x^2 - 3 \neq 0$$

$$x \neq \pm \sqrt{3}$$

Dom f : $x \neq \sqrt{3}, -\sqrt{3}$

Sec. 3, linear func - cost, revenue, price, profit

Cost: $C(x)$

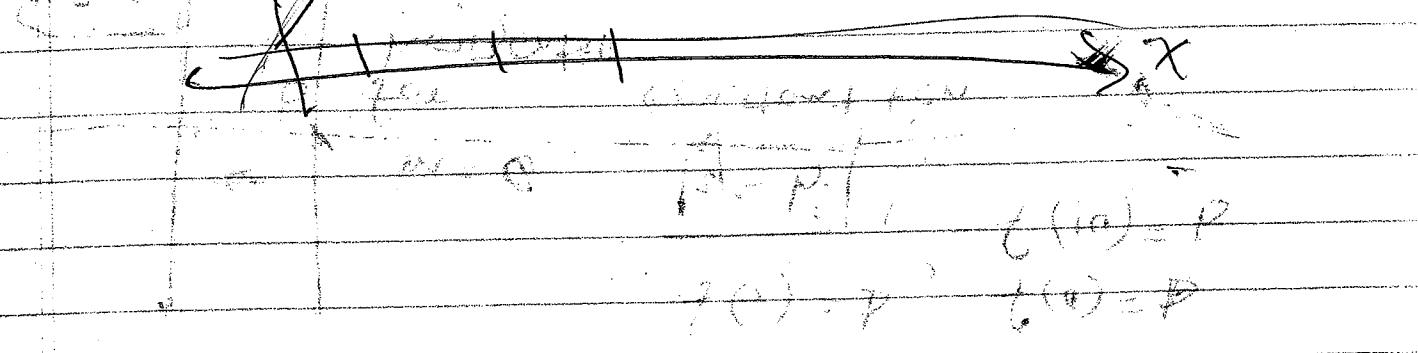
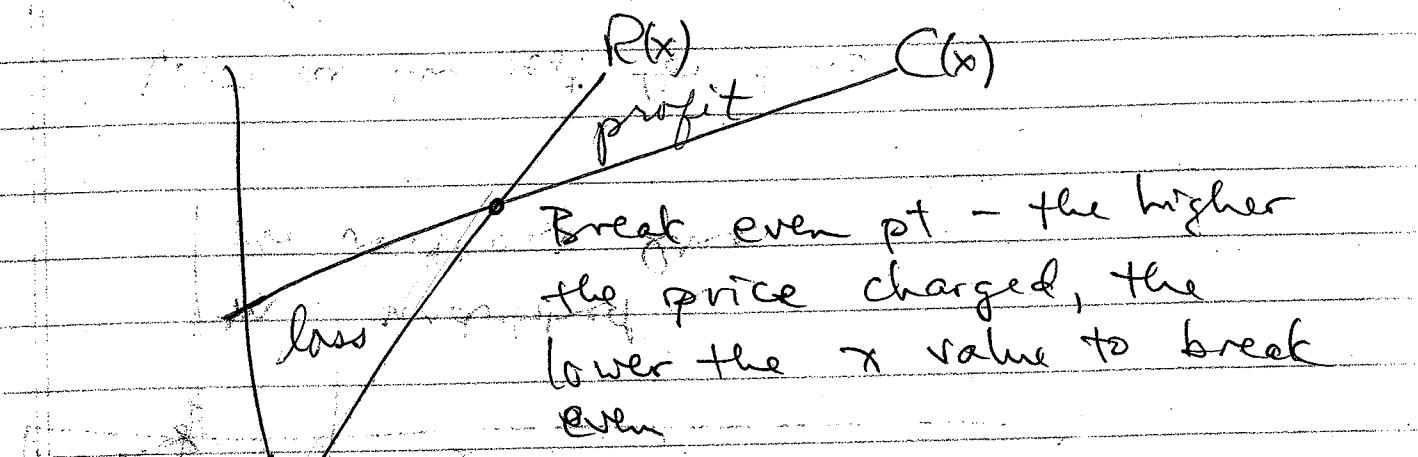
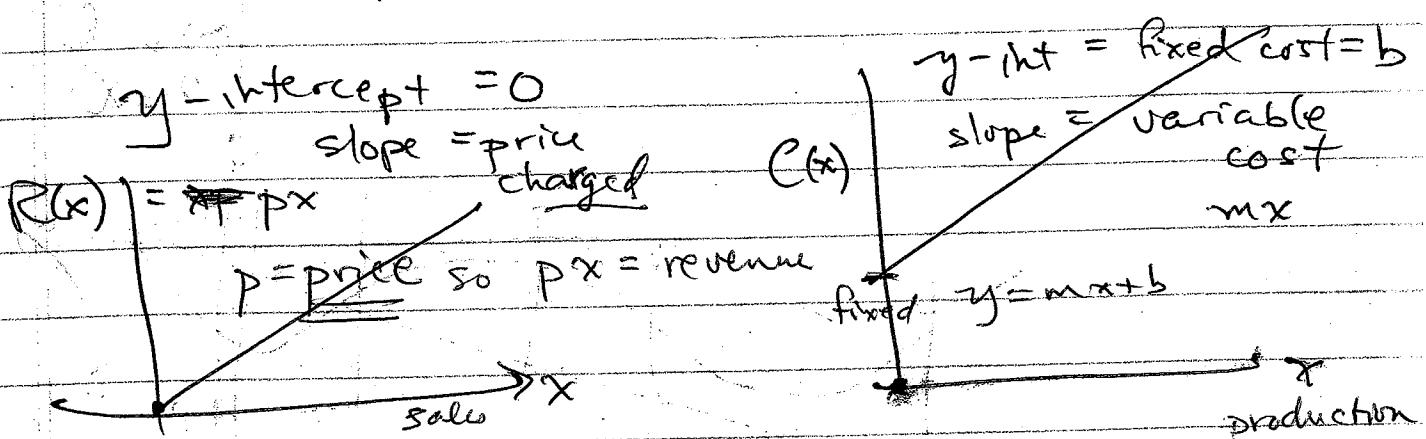
Supply/demand: x (either manufactured amt or amt sold)

Price: p

Revenue: px (price \times demand)

Profit: ~~Cost~~ ~~Revenue~~ Revenue - Cost = $P(x)$

Break-even point: $R(x) = C(x)$, i.e. $P(x) = 0$



$$f(x) = b, \quad f(0) = b$$

$$f(10) = b$$

$$m = 0$$

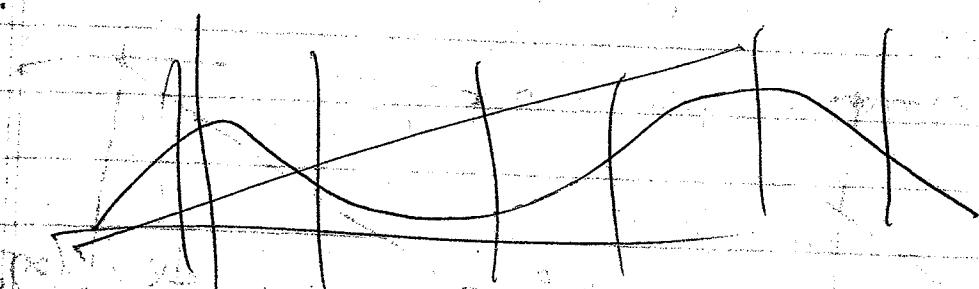
$$ky = b$$

b for
horizontal
line

$$x = a$$

m is undefined
for vertical line.

Vertical line test for fns:



$$x = 0$$

y-axis

$$(0,0)$$

$$f(x) = y = 0, \quad x\text{-axis}$$

III. Any root func, like $f(x) = \sqrt{x}$ has a domain restriction on the radicand that it be nonnegative

Ex $f(x) = \sqrt{x}, x \geq 0, [0, \infty)$

Ex $g(x) = \sqrt[3]{x+2}, x+2 \geq 0, x \geq -2$

Ex $f(x) = \sqrt[3]{x}, x \in \mathbb{R}$, since we may take an odd root of a negative no.

Odd roots have no restriction on domain

"Solving for roots of a func" means finding those values where $f(x) = 0$. Often, we're interested in the roots of quadratic eqns.

Ex $\rightarrow y = x^2 - 6x - 7$ has roots at $x =$

$$y = (x-7)(x+1) = 0, (x=7, -1)$$

factor

Ex $f(x) = \frac{x+4}{x^2-9}$ the roots are where the numerator = 0, you can ignore the denominator, but be careful

$$f(x) = 0 \text{ where } x+4 = 0 \text{ or } (x = -4) \text{ root}$$

$$\text{Dom } f(x): x \neq \pm 3$$

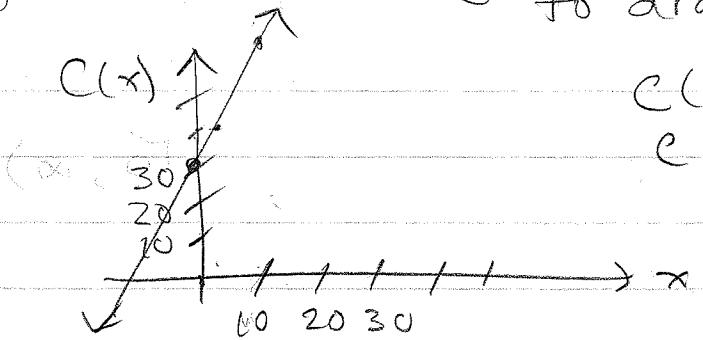
Graphing linear func. - To graph a linear func, it suffices to know the y -intercept and the slope:

Ex $C(x) = 5x + 30$

$$\text{slope} = \frac{5}{1}$$

$$\text{when } x=0, y=30$$

$$C(x) = 5x + 30$$



You need an appropriate scale to draw a decent graph

$$C(0) = 30$$

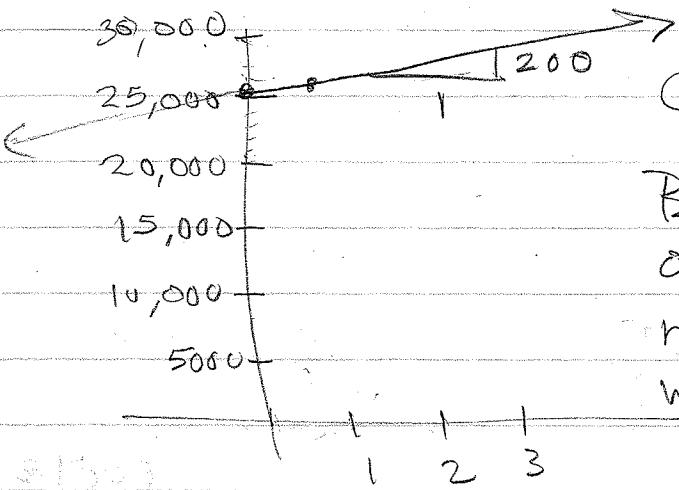
$$C(10) = 80$$

Cost fn. $C(x)$

Another ex, the book's Ex 3.4. i) where x = quantity produced

$$\rightarrow C(x) = 200x + 25,000 \quad \text{Here, } m = 200, b = 25,000$$

The scale on the vertical axis will need to be larger than the horizontal axis to accomodate the y-intercept; $(0, 25,000)$



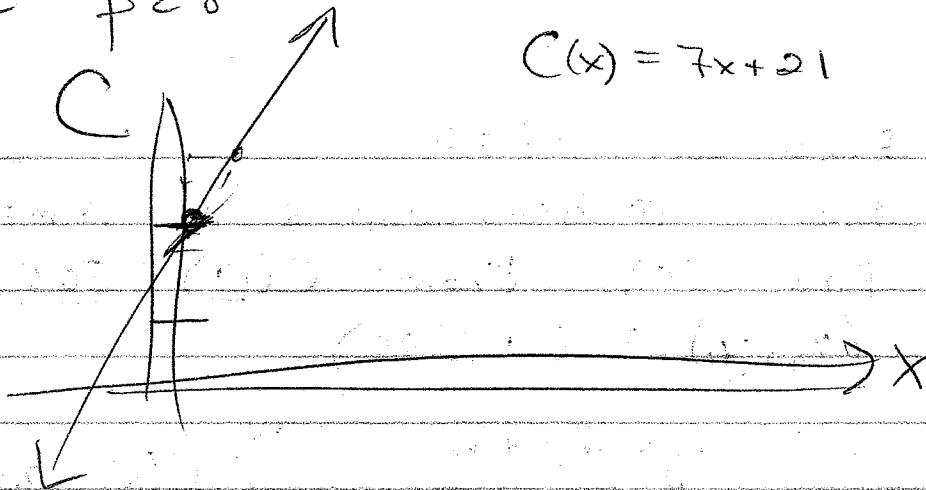
Clearly not a one-to-one scale

But the goal is to make sense of the magnitude of the slope, regardless of the y-intercept, which is the starting point.

The slope of a cost fn. $C(x)$ is the variable

#12 p28

$$C(x) = 7x + 21$$



$$R(x) = 14x = px$$

$$P(x) = R(x) - C(x)$$

$$= 14x - (7x + 21)$$

$$= 14x - 7x - 21$$

$$\boxed{\text{Profit} = \underline{7x - 21} = 0}$$

$$P(x) \uparrow$$

$$7x - 21 = 0$$

$$x = 3 \text{ desserts}$$

(3, 0)

x sold

$$P(x) = 500 = 7x - 21 \rightarrow 521 = 7x$$

$$\cancel{479 = 7x}$$

$$\cancel{7} \cancel{x}$$

$$x = \frac{521}{7}$$

14.2857 x

$$x = 74$$

#1 Fixed cost - \$150

Know 10 items cost \$300 to make

Find $C(x)$. Know $C(10) = 300$

Pt: $(x_1, y_1) = (10, 300)$

$$y = mx + b$$

$$C(x) = \text{fun}$$

$$C(0) = 150 = b$$

$$C(x) = mx + 150$$

unhelpful

$$\text{But } y - y_1 = m(x - x_1)$$

$$y = mx + b$$

$$150 = m(0) + b \quad b = 150$$

$$(x, y) = (0, 150) \quad m = ? \quad = \frac{300 - 150}{10 - 0} = 15$$

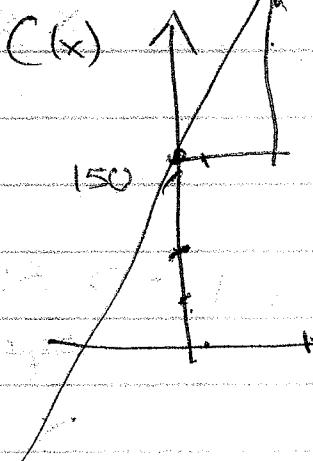
$$(10, 300)$$

$$C(x) = 15x + 150$$

(MC)

linear cost

Def Marginal cost - for a ~~linear~~ cost fun,
MC = slope, representing the cost of
~~each~~ producing a single item.



Math 220 - Friday Sept 4

notes were emailed & will be
Scan of ~~key~~ posted for next week's updated
website.

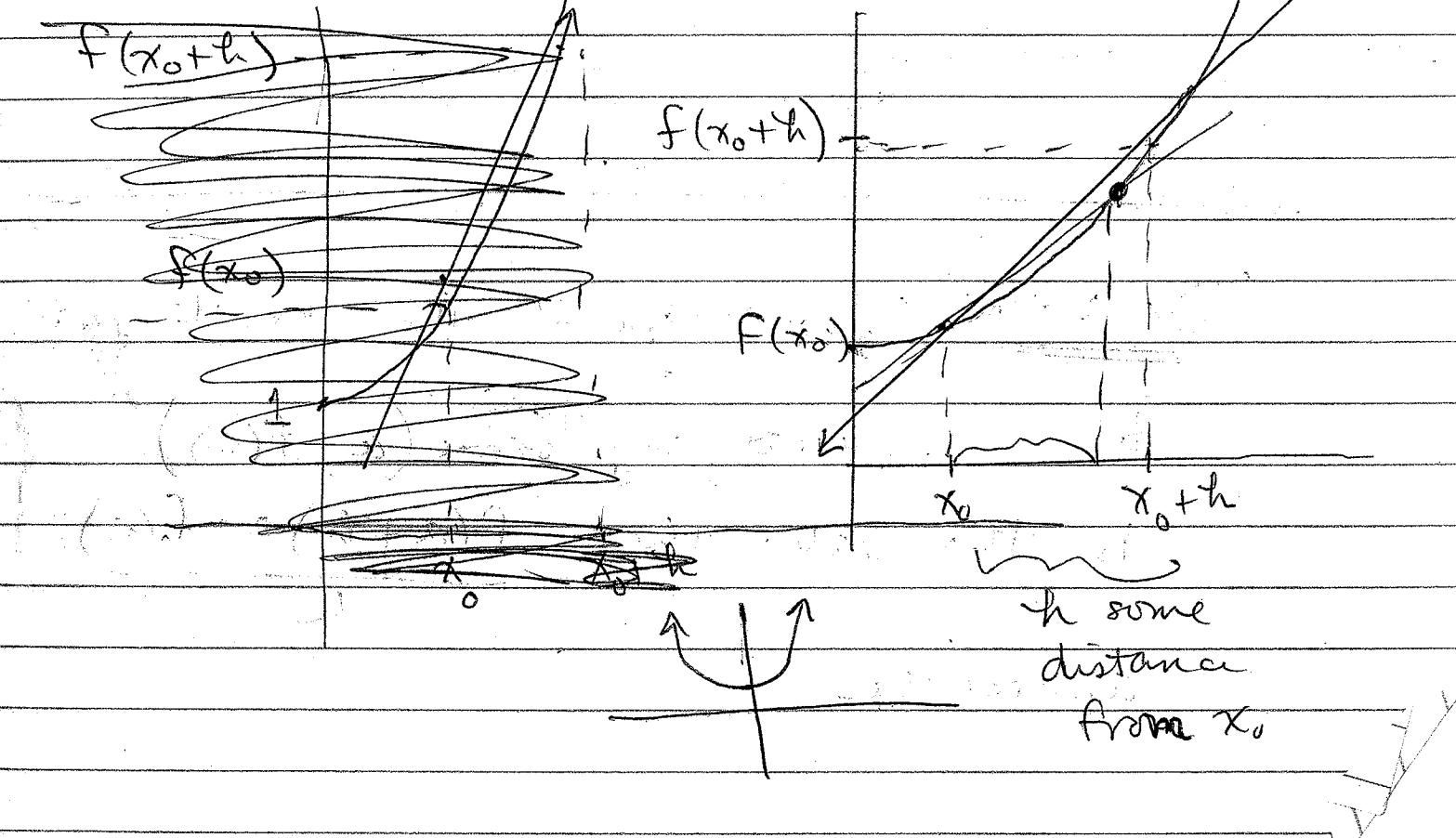
HW for week 1 - complete, annotated solutions,
will also be posted.

Today - we skip to Section 6, slope of a line.

We'll start with a basic non-linear fun:

$$y = f(x) = x^2 + 1$$

Sketch the graph of this parabola & focus on
the first quadrant.

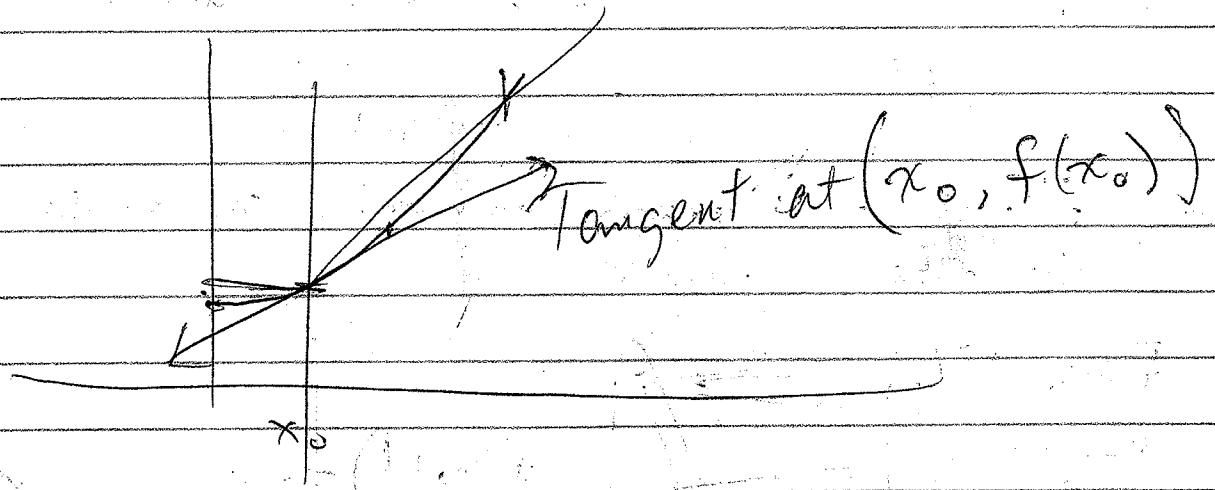


Slope of Secant =

$$\frac{x_0^2 + 2x_0 h + h^2 + f - (x_0^2 + f)}{x_0 + h - x_0}$$

$$\text{slope} = \frac{2x_0 h + h^2}{h} = \boxed{\frac{2x_0 + h}{1}}$$

Because we've skipped limits for now, let's say "as h gets smaller, slope gets closer to $2x_0$.



Definition. The slope of the line tangent

to a curve at any point $(a, f(a))$

is the value the $\frac{f(a+h) - f(a)}{h}$

Approaches

$$\#7b) MC = \$100 = m$$

$$x = 10, C(x) = 2237$$

$$(x_1, y_1) = (10, 2237)$$

$$y - y_1 = m(x - x_1)$$

$$y - 2237 = 100(x - 10) \quad (100)(x-10)$$

$$C(x) = y = 100x + 1237$$

Skip to p. 69 Ch. 7.

$$f(x) = x^2 + 1$$

$$f(x_0) = x_0^2 + 1$$

~~for~~

where x_0 is a given value
of x

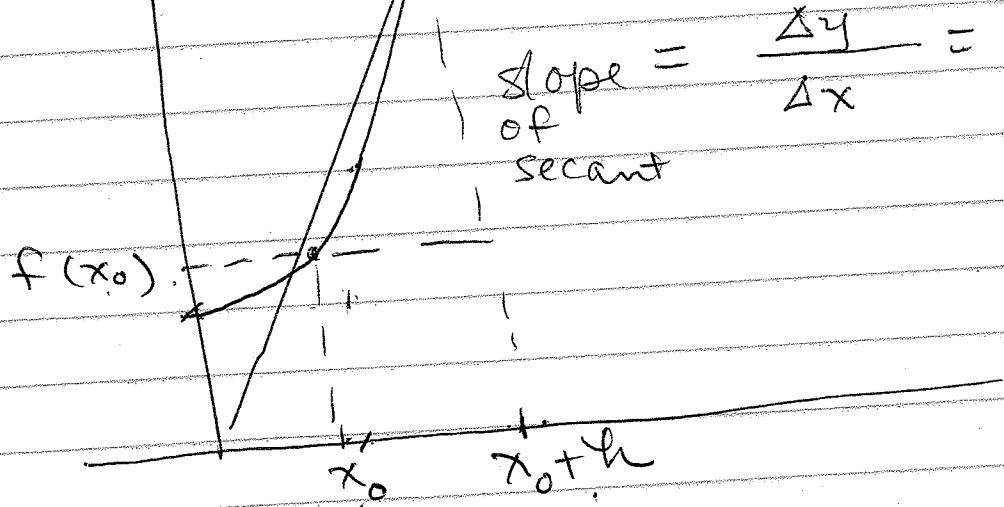
So if $x_0 = 9$

$$\rightarrow f(9) = 9^2 + 1 = 82$$

$$f(x) = x^2 + 1 \quad f(x_0 + h) = (x_0 + h)^2 + 1$$

$$f(x_0 + h) = x_0^2 + 2x_0h + h^2 + 1$$

$$\text{slope of secant} = \frac{\Delta y}{\Delta x} = \frac{x_0^2 + 2x_0h + h^2 + 1 - (x_0^2 + 1)}{x_0 + h - x_0}$$



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- Linear funcs + equations of them

$$y = mx + b$$

$$y - y_1 = m(x - x_1) \quad (x_1, y_1)$$

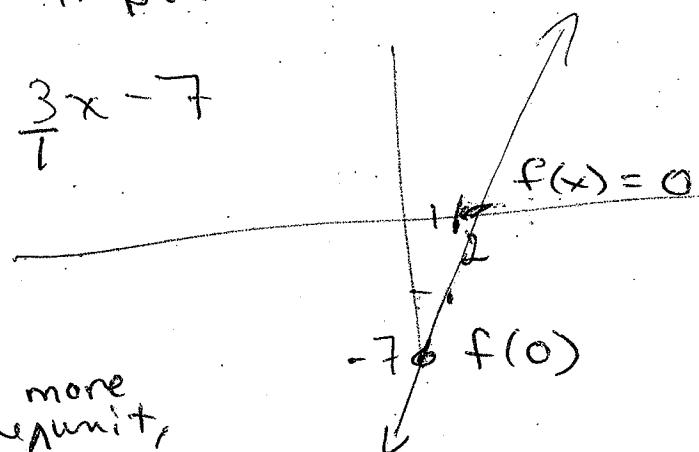
Given

Any time ~~you~~ you see a subscripted variable, it means you have a value either given or implied to be set.

Ex $f(x) = 3x - 7$

If $f(x)$ were
a cost func, then

slope is the cost more
of producing one unit,
i.e., marginal cost

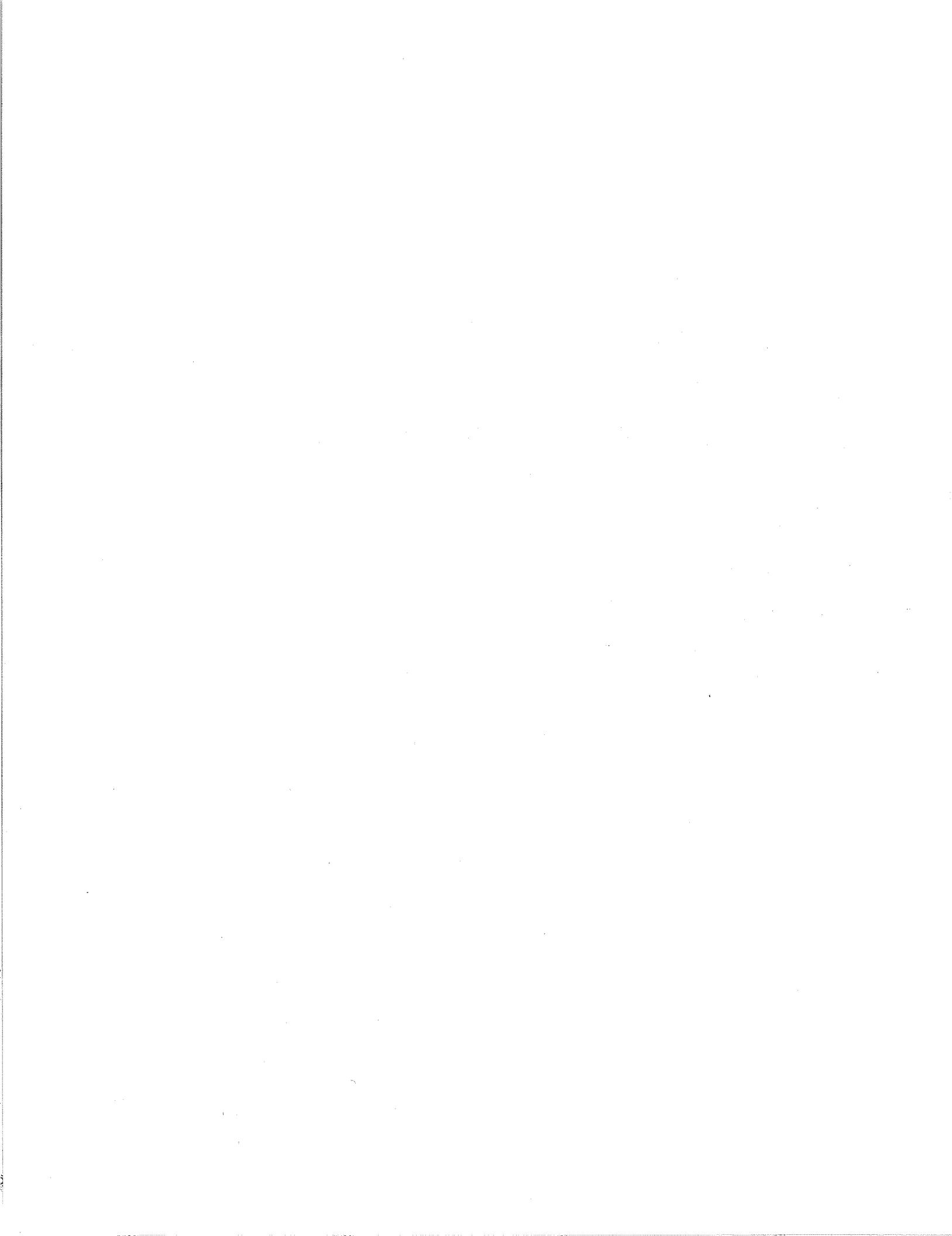


$$C(x) = \$3x + 40 \rightarrow \begin{matrix} \text{fixed} \\ \text{cost} \end{matrix}$$

\swarrow
slope
- variable cost

~~of production~~
of production

(turning on
the lights)



$$C(x), R(x), P(x)$$

↓ ↓ → profit

$$C(x) = \text{cost fcn} \quad p \cdot x \text{ revenue fcn}$$

$$R(x) - C(x) = P(x)$$

x = units produced or sold

p = price per item

Break-even production: Set $P(x) = 0$
that is, $R(x) = C(x)$

Note: Sometimes you want to find production amt x to break even;

Sometimes you're given a production level (given x) and you need to find p price to set to break even + then increase it.

80

- Non-linear fns and determined the marginal cost from the slope of the tangent line to the graph at some coordinate x_0 or $x=a$

Why slope? \rightarrow marginal cost function

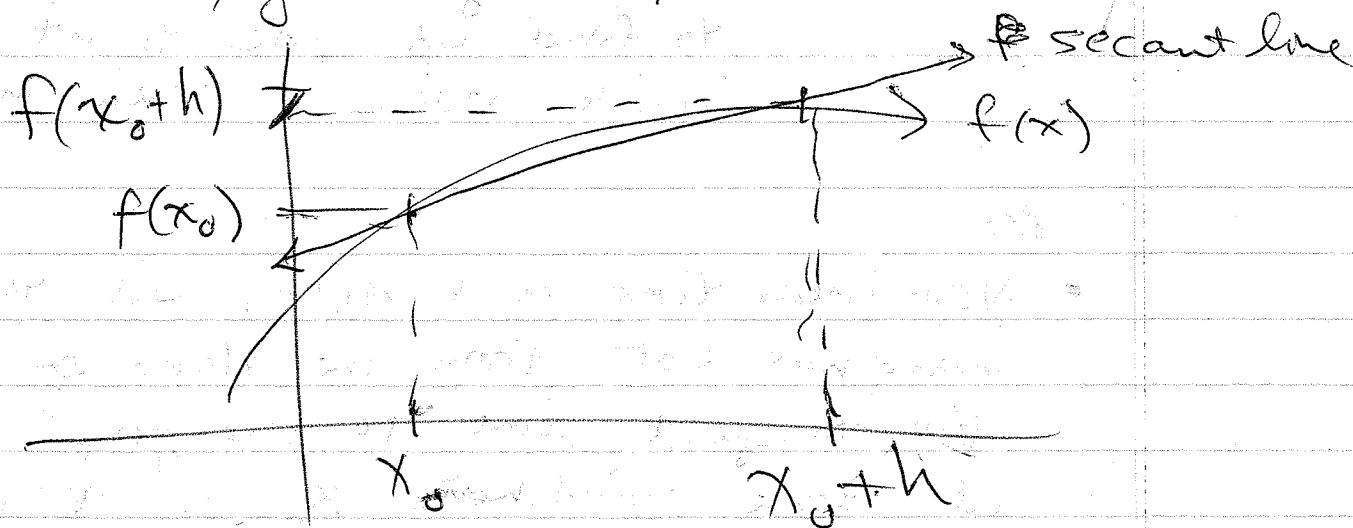
slope
of the tangent

Marginal cost denominator, straight line or derived slope of tangent line, is = 1 for "one more unit"

By "derived," we mean the expression you arrive at by inspecting, calculating the difference quotient for fcn $f(x)$ at an x value, ~~whether~~ given or ~~variables~~ⁱⁿ general.

The Process

Given $f(x)$, find $\frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0}$ Slope of secant line as h , which is a distance right from x_0 , gets smaller, even to zero.



The diff. quotient gives the marginal cost of producing the $x_0 + 1$ unit.

Sec 7 p. 72

1. a

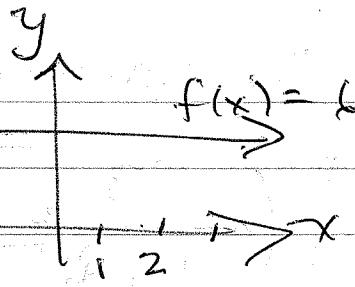
$$f(x) = 6$$

for all x

$m = 0$ is the

slope for every tangent

$m = 2$ ~~at~~ at $x=2$, at all x in fact



b. $f(x) = 7 - 5x$, at $x=12$

$m = -5$. at all x , inc. $x=12$

c)

$$f(x) = \frac{3}{x}$$

$$y = \frac{\text{Const}}{x}$$

Find slope at $x = \frac{1}{2} = x_0$

D.Q. $\frac{f(x_0+h) - f(x_0)}{(x_0+h) - x_0} = \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h}$

diff.
quot.

$$f\left(\frac{1}{2}+h\right) = \frac{3}{\frac{1}{2}+h}, \quad f\left(\frac{1}{2}\right) = \frac{3}{\frac{1}{2}} = 6$$

So D.Q. = $\frac{\frac{3}{\frac{1}{2}+h} - 6}{h}$ But if you
let $h \rightarrow 0$ what happens
to denom?

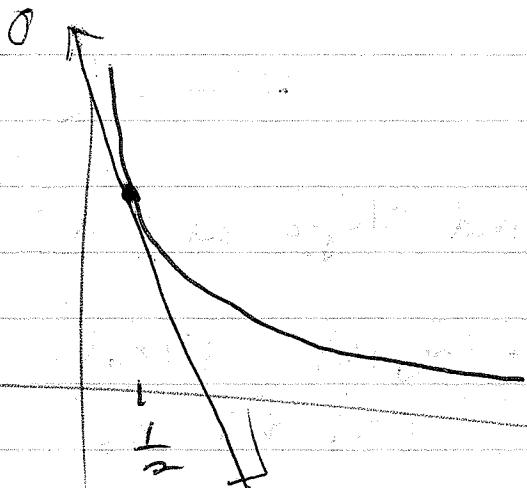
You get something ∞
zero

Do some simplifying via algebra

$$\frac{\left(\frac{1+h}{2}\right) \left(3\right) - \left(\frac{6}{1+h}\right)}{\left(\frac{1+h}{2}\right)} \cdot \frac{h}{h} \quad \text{LCD} = 1+h$$

$$= \frac{3 - 3 - 6h}{\frac{h}{2} + h^2} = \frac{-6h}{\frac{h}{2} + h^2}$$

$$= \frac{-6}{\frac{1+h}{2}} \xrightarrow[h \rightarrow 0]{\text{Let}} \frac{-6}{1/2} = -12$$

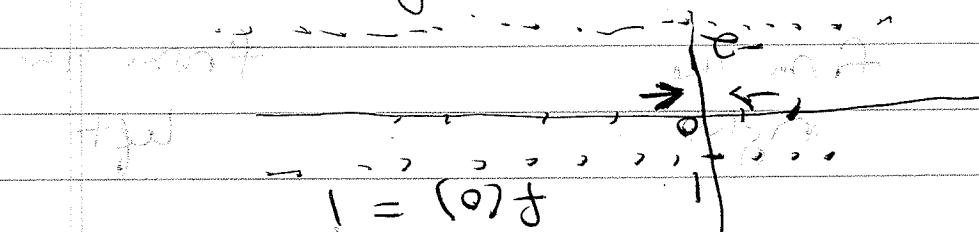


Section 6 | Limits and the Derivative

For our purposes, "unit" will be defined loosely, rather than using ^{the} mathematical precision of Greek symbols + their differences.

So, to say "the limit of a function $f(x)$ is a number L " means

Def "The limit of $f(x)$ is L as x approaches a
means that as x gets ~~arbitrarily~~^{very} close to a , $f(x)$ gets arbitrarily close to L .



you can now group your own (now no longer
is unique) or group them. Go to the right

(Bottom) F_1 = $\text{f}_{\text{rect}} \text{t}_1$; d_{bottom} that respect
fractional, F_2 , F_3 cannot be shown by fraction
that is, a ~~numerating~~ decimal

~~forward~~ ~~forward~~ ~~forward~~

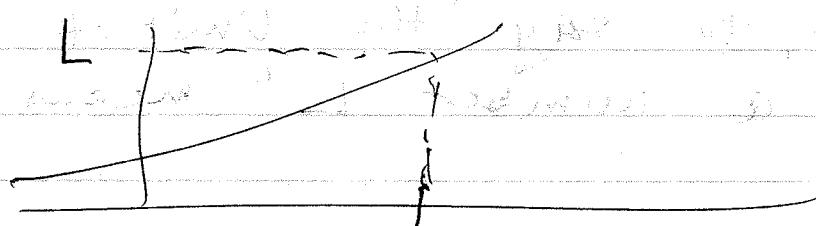
$\theta = \pi - \alpha$ and $\beta = \pi - \theta$

$$R = \bigcup_{i=1}^n R_i$$

Limit of function $f(x) = L$ as $x \rightarrow a$

means
to left of a then ~~function~~ \rightarrow no \neq

As $x \rightarrow a^-$ from the left, $\lim f(x) = L$
As $x \rightarrow a^+$ from the right, $\lim f(x) = L$



$\rightarrow \leftarrow$

so $x \rightarrow a$ to find $\lim f(x)$

$\lim_{x \rightarrow a^+} f(x) = L = \lim_{x \rightarrow a^-} f(x)$

from the
right

from the
left

Ex for $f(x) = x^2 - 4$

$\lim_{x \rightarrow 2^+} (x^2 - 4) = ? = 0$

for $x = 2.2$ $x = 1.8$

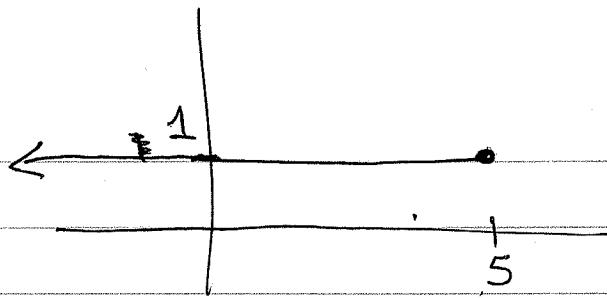
$x = 2.1$ $x = 1.9$

$x = 2.01$ $x = 1.99$

$x = 2.001$ $x = 1.999$

and $\lim_{x \rightarrow 2^+} (x^2 - 4) = ? = 0$

Ex 6.4



$$f(x) = \begin{cases} 1 & x \leq 5 \\ -2 & x > 5 \end{cases}$$

$$\lim_{x \rightarrow 5^-} f(x) = 1, \quad \lim_{x \rightarrow 5^+} f(x) = -2$$

Because $LHL \neq RHL$

(left-hand limit) (right-hand limit)

we say $f(x)$ has no limit as $x \rightarrow 5$.

or $\lim_{x \rightarrow 5} f(x)$ DNE.

So if $LHL \neq RHL$, the limit DNE.

If the limit does exist, then the $LHL = RHL$

$$S = S(\alpha^2) \text{ and } T = T(\alpha^2) \text{ satisfy}$$

Scutellaria *Scutellaria*

1484 + 111 = 1595

It took us half an hour to get out

First (x) 2nd

1960-08-14, 1960-08-15

With this agreement, the two sides worked out a 70%

Limits - Continued

Recall that the statement:

$$\lim_{x \rightarrow a} f(x) = L$$

means that as x gets closer to a , $f(x)$ gets arbitrarily close^{to L} (as close as you like without actually being L)

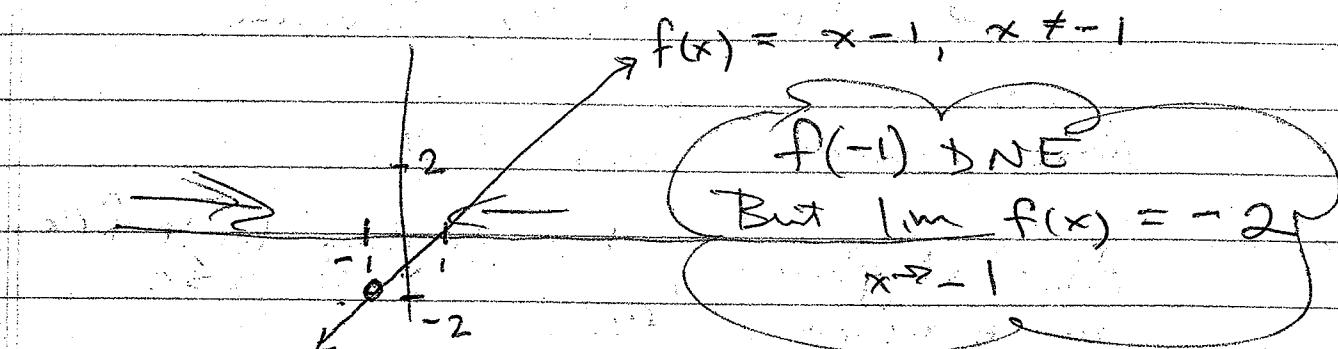
So, for the function $f(x) = \frac{x^2 - 1}{x + 1}$

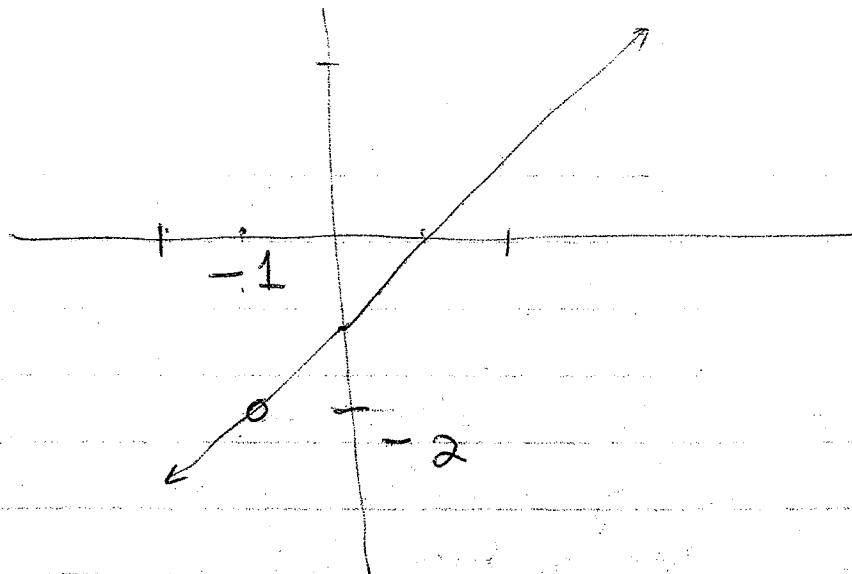
$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{(x+1)} = \lim_{x \rightarrow -1} (x-1) = -2$$

Note that the domain of f is all reals except -1 . $f(-1)$ does not exist.

But it isn't necessary for $f(a)$ to exist to have $\lim_{x \rightarrow a} f(x)$ exist.





Evaluating limits requires we apply the "theorem" (p. 63) and do some algebra so the form we have as $x \rightarrow a$ is not something that is "indeterminate" — that is, something like $\frac{0}{0}$ or $(\frac{5}{0})$.

Examples 6.8 + 6.9 address this technique.

First, a well-behaved function, polynomial

$$\lim_{x \rightarrow b} P(x), \text{ where } P(x) = q_n x^n + q_{n-1} x^{n-1} + \dots + q_1 x + q_0$$

is simply $P(b)$, a value.

Now consider 6.8 + 6.9.

At first glance, try finding $\lim_{x \rightarrow a} f(x)$ by evaluating $f(a)$