In Problems 19–30, consider the given quadratic function f.

Roots

(a) Find all intercepts of the graph of f.
(b) Express the function f in standard form.
(c) Find the vertex and axis of symmetry.
(d) Sketch the graph of f.

19. $f(x) = x(x + 5)$
20. $f(x) = -x^2 + 4x$
21. $f(x) = (3 - x)(x + 1)$
22. $f(x) = (x - 2)(x - 6)$
23. $f(x) = x^2 - 3x + 2$
24. $f(x) = -x^2 + 6x - 5$
25. $f(x) = 4x^2 - 4x + 3$
26. $f(x) = -x^2 + 6x - 10$
27. $f(x) = -\frac{1}{2}x^2 + x + 1$
28. $f(x) = x^2 - 2x - 7$
29. $f(x) = x^2 - 10x + 25$
30. $f(x) = -x^2 + 6x - 9$

In Problems 31 and 32, find the maximum or the minimum value of the function f. Give the range of the function f.

31. $f(x) = 3x^2 - 8x + 1$
32. $f(x) = -2x^2 - 6x + 3$

In Problems 33–36, find the largest interval on which the function f is increasing and the largest interval on which f is decreasing.

33. $f(x) = \frac{1}{3}x^2 - 25$
34. $f(x) = -(x + 10)^2$
35. $f(x) = -2x^2 - 12x$
36. $f(x) = x^2 + 8x - 1$

Parabola Transformations

In Problems 37–42, describe in words how the graph of the given function f can be obtained from the graph of $y = x^2$ by rigid or nonrigid transformations.

37. $f(x) = (x - 10)^2$
38. $f(x) = (x + 6)^2$
39. $f(x) = -\frac{1}{3}(x + 4)^2 + 9$
40. $f(x) = 10(x - 2)^2 - 1$
41. $f(x) = (-x - 6)^2 - 4$
42. $f(x) = -(1 - x)^2 + 1$

(I'll give you others on Monday - see our book for non-parabola transforms.

In Problems 43–48, the given graph is the graph of $y = x^2$ shifted/reflected in the xy-plane. Write an equation of the graph.

43.

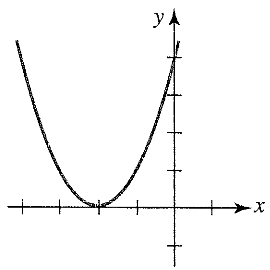


FIGURE 3.3.12 Graph for Problem 43

44.

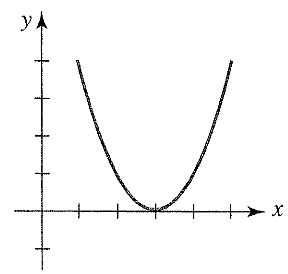


FIGURE 3.3.13 Graph for Problem 44

47.

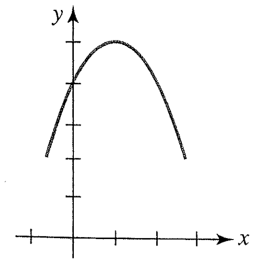


FIGURE 3.3.16 Graph for Problem 47

In Problems 49 and 50, find
the given conditions.

49. f has the values $f(0) = 5$
50. Graph passes through (2,

In Problems 51 and 52, find a q
that satisfies the given conditi

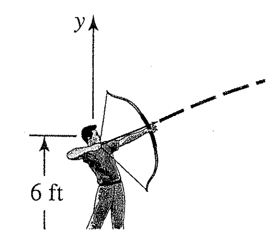
51. The vertex of the graph o
52. The maximum value of f i

In Problems 53–56, sketch the
of the given functions. Find th

53. $y = -x + 4$, $y = x^2 + 2$
55. $y = x^2 + 2x + 2$, $y = -x$
57. (a) Express the square of
$y = 2x$ to the point (
(b) Use the function in p

Miscellaneous Applicatio

58. **Shooting an Arrow** As s
angle with the horizontal
$y = ax^2 + x + c$. Use th
6 ft and travels a horizont
What is the maximum hei



15.

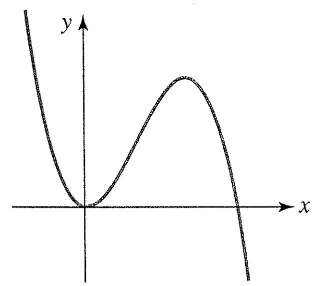


FIGURE 4.1.14 Graph for Problem 15

16.

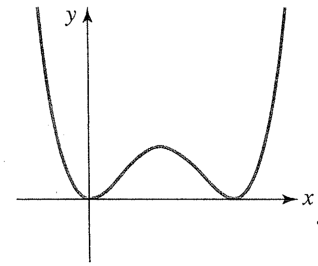


FIGURE 4.1.15 Graph for Problem 16

17.

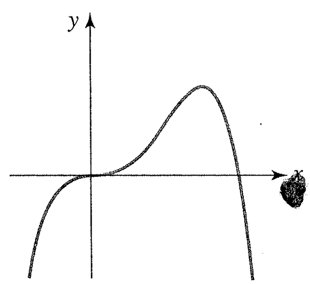


FIGURE 4.1.16 Graph for Problem 17

18.

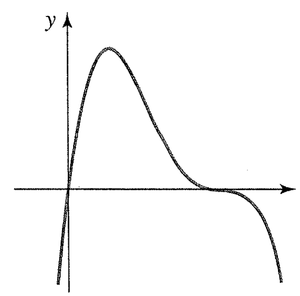


FIGURE 4.1.17 Graph for Problem 18

In Problems 19–40, proceed as in Example 2 and sketch the graph of the given polynomial function f.

19. $f(x) = x^3 - 4x$
20. $f(x) = 9x - x^3$
21. $f(x) = -x^3 + x^2 + 6x$
22. $f(x) = x^3 + 7x^2 + 12x$
23. $f(x) = (x + 1)(x - 2)(x - 4)$
24. $f(x) = (2 - x)(x + 2)(x + 1)$
25. $f(x) = x^4 - 4x^3 + 3x^2$
26. $f(x) = x^2(x - 2)^2$
27. $f(x) = (x^2 - x)(x^2 - 5x + 6)$
28. $f(x) = x^2(x^2 + 3x + 2)$
29. $f(x) = (x^2 - 1)(x^2 + 9)$
30. $f(x) = x^4 + 5x^2 - 6$
31. $f(x) = -x^4 + 2x^2 - 1$
32. $f(x) = x^4 - 6x^2 + 9$
33. $f(x) = x^4 + 3x^3$
34. $f(x) = x(x - 2)^3$
35. $f(x) = x^5 - 4x^3$
36. $f(x) = (x - 2)^5 - (x - 2)^3$
37. $f(x) = 3x(x + 1)^2(x - 1)^2$
38. $f(x) = (x + 1)^2(x - 1)^3$
39. $f(x) = -\frac{1}{2}x^2(x + 2)^3(x - 2)^2$
40. $f(x) = x(x + 1)^2(x - 2)(x - 3)$

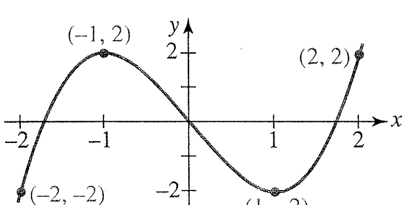


FIGURE 4.1.18 Graph for Problem 41

41. The graph of $f(x) = x^3 - 3x$ is given in FIGURE 4.1.18.
(a) Use the figure to obtain the graph of $g(x) = f(x) + 2$.
(b) Using only the graph obtained in part (a) write an equation, in *factored* form,

46. Consider the polynomial fun
positive integer. For what val
cross, the x-axis at $(2, 0)$?
47. Consider the polynomial func
positive integer. For what valu
48. Consider the polynomial func
are positive integers.
(a) For what values of m doe
(b) For what values of n does

Miscellaneous Applications

49. **Constructing a Box** An open
cardboard by cutting a square
sides. See FIGURE 4.1.20. If the
the volume of the resulting bo

$$V(x) =$$

Sketch the graph of $V(x)$ for x
50. **Another Box** In order to hold
or some other fastener at the co
be made by cutting out a squar
piece of cardboard, cutting on
shown in FIGURE 4.1.21. Find a p
the resulting box if the original
graph of $V(x)$ for $x > 0$.

For Discussion

51. Examine Figure 4.1.5. Then dis
functions that have no real zero
52. Suppose a polynomial function
behavior that its graph goes dow
$x \to \infty$. Discuss possible equati

Calculator/Computer Problems

In Problems 53 and 54, use a graphi
nomial function on the indicated inte

53. $f(x) = -(x - 8)(x + 10)^2$;
54. $f(x) = (x - 5)^2(x + 5)^2$; [−

sing Synthetic Division to Evaluate a Function

find $f(2)$ for

$^6 + 4x^5 + x^4 - 8x^3 - 6x^2 + 9.$

division to find the remainder r in the division of f by
n all the coefficients in $f(x)$, including 0 as the coeffi

1	−8	−6	0	9
4	−6	−28	−68	−136
3	−14	−34	−68	$\boxed{-127} = r$

ng Synthetic Division to Evaluate a Function

$f(x) = x^3 - 7x^2 + 13x - 15$ at $x = 5$.

ion

−7	13	−15
5	−10	15
−2	3	$\boxed{0} = r$

5) = 0 shows that 5 is a zero of the given function
ally that f is evenly divisible by the linear polyno
5 is a factor of f. The synthetic division shows that
ivalent to

$x - 5)(x^2 - 2x + 3).$

er explore the use of the Division Algorithm and
ling zeros and factors of a polynomial function.

5. $f(x) = 2x^3 + 4x^2 - 3x + 5;\quad g(x) = (x + 2)^2$
6. $f(x) = x^3 + x^2 + x + 1;\quad g(x) = (2x + 1)^2$
7. $f(x) = 27x^3 + x - 2;\quad g(x) = 3x^2 - x$
8. $f(x) = x^4 + 8;\quad g(x) = x^3 + 2x - 1$
9. $f(x) = 6x^5 + 4x^4 + x^3;\quad g(x) = x^3 - 2$
10. $f(x) = 5x^6 - x^5 + 10x^4 + 3x^2 - 2x + 4;\quad g(x) = x^2 + x - 1$

In Problems 11–16, proceed as in Example 2 and use the Remainder Theorem to find r when $f(x)$ is divided by the given linear polynomial.

11. $f(x) = 2x^2 - 4x + 6;\quad x - 2$
12. $f(x) = 3x^2 + 7x - 1;\quad x + 3$
13. $f(x) = x^3 - 4x^2 + 5x + 2;\quad x - \frac{1}{2}$
14. $f(x) = 5x^3 + x^2 - 4x - 6;\quad x + 1$
15. $f(x) = x^4 - x^3 + 2x^2 + 3x - 5;\quad x - 3$
16. $f(x) = 2x^4 - 7x^2 + x - 1;\quad x + \frac{3}{2}$

Long ÷

In Problems 17–22, proceed as in Example 3 and use the Remainder Theorem to find $f(c)$ for the given value of c.

17. $f(x) = 4x^2 - 10x + 6;\quad c = 2$
18. $f(x) = 6x^2 + 4x - 2;\quad c = \frac{1}{4}$
19. $f(x) = x^3 + 3x^2 + 6x + 6;\quad c = -5$
20. $f(x) = 15x^3 + 17x^2 - 30;\quad c = \frac{1}{5}$
21. $f(x) = 3x^4 - 5x^2 + 20;\quad c = \frac{1}{2}$
22. $f(x) = 14x^4 - 60x^3 + 49x^2 - 21x + 19;\quad c = 1$

In Problems 23–32, use synthetic division to find the quotient $q(x)$ and remainder $r(x)$ when $f(x)$ is divided by the given linear polynomial.

23. $f(x) = 2x^2 - x + 5;\quad x - 2$
24. $f(x) = 4x^2 - 8x + 6;\quad x - \frac{1}{2}$
25. $f(x) = x^3 - x^2 + 2;\quad x + 3$
26. $f(x) = 4x^3 - 3x^2 + 2x + 4;\quad x - 7$
27. $f(x) = x^4 + 16;\quad x - 2$
28. $f(x) = 4x^4 + 3x^3 - x^2 - 5x - 6;\quad x + 3$
29. $f(x) = x^5 + 56x^2 - 4;\quad x + 4$
30. $f(x) = 2x^6 + 3x^3 - 4x^2 - 1;\quad x + 1$
31. $f(x) = x^3 - (2 + \sqrt{3})x^2 + 3\sqrt{3}x - 3;\quad x - \sqrt{3}$
32. $f(x) = x^8 - 3^8;\quad x - 3$

More Long ÷

In Problems 33–38, use synthetic division and the Remainder Theorem to find $f(c)$ for the given value of c.

33. $f(x) = 4x^2 - 2x + 9;\quad c = -3$
34. $f(x) = 3x^4 - 5x^2 + 27;\quad c = \frac{1}{2}$
35. $f(x) = 14x^4 - 60x^3 + 49x^2 - 21x + 19;\quad c = 1$

If $P(r)=0$ then $x-r$ is a factor. Determine if this is the case. Then find other zeros if it is.

4.3 Exercises

Answers to selected odd-numbered problems begin on page ANS-12.

In Problems 1–6, determine whether the indicated real number is a zero of the given polynomial function f. If yes, find all other zeros and then give the complete factorization of $f(x)$.

1. 1; $f(x) = 4x^3 - 9x^2 + 6x - 1$

2. $\frac{1}{2}$; $f(x) = 2x^3 - x^2 + 32x - 16$

3. 5; $f(x) = x^3 - 6x^2 + 6x + 5$

4. 3; $f(x) = x^3 - 3x^2 + 4x - 12$

5. $-\frac{2}{3}$; $f(x) = 3x^3 - 10x^2 - 2x + 4$

6. -2; $f(x) = x^3 - 4x^2 - 2x + 20$

In Problems 7–10, verify that each of the indicated numbers are zeros of the given polynomial function f. Find all other zeros and then give the complete factorization of $f(x)$.

7. $-3, 5$; $f(x) = 4x^4 - 8x^3 - 61x^2 + 2x + 15$

8. $\frac{1}{4}, \frac{3}{2}$; $f(x) = 8x^4 - 30x^3 + 23x^2 + 8x - 3$

9. $1, -\frac{1}{3}$ (multiplicity 2); $f(x) = 9x^4 + 69x^3 - 29x^2 - 41x - 8$

10. $-\sqrt{5}, \sqrt{5}$; $f(x) = 3x^4 + x^3 - 17x^2 - 5x + 10$

In Problems 11–16, use synthetic division to determine whether the indicated linear polynomial is a factor of the given polynomial function f. If yes, find all other zeros and then give the complete factorization of $f(x)$.

11. $x - 5$; $f(x) = 2x^2 + 6x - 25$

12. $x + \frac{1}{2}$; $f(x) = 10x^2 - 27x + 11$

13. $x - 1$; $f(x) = x^3 + x - 2$

14. $x + \frac{1}{2}$; $f(x) = 2x^3 - x^2 + x + 1$

15. $x - \frac{1}{3}$; $f(x) = 3x^3 - 3x^2 + 8x - 2$

16. $x - 2$; $f(x) = x^3 - 6x^2 - 16x + 48$

In Problems 17–20, use division to show that the indicated polynomial is a factor of the given polynomial function f. Find all other zeros and then give the complete factorization of $f(x)$.

Divide as x^2-3x+2 into $f(x)$ →

17. $(x - 1)(x - 2)$; $f(x) = x^4 - 3x^3 + 6x^2 - 12x + 8$

18. $x(3x - 1)$; $f(x) = 3x^4 - 7x^3 + 5x^2 - x$

19. $(x - 1)^2$; $f(x) = 2x^4 + x^3 - 5x^2 - x + 3$

20. $(x + 3)^2$; $f(x) = x^4 - 4x^3 - 22x^2 + 84x + 261$

In Problems 21–26, verify that the indicated complex number is a zero of the given polynomial function f. Proceed as in Example 7 to find all other zeros and then give the complete factorization of $f(x)$.

21. $2i$; $f(x) = 3x^3 - 5x^2 + 12x - 20$

22. $\frac{1}{2}i$; $f(x) = 12x^3 + 8x^2 + 3x + 2$

In Problems 33–36, find the zeros o
of each zero.

33. $f(x) = x(4x - 5)^2(2x - 1)^3$

35. $f(x) = (9x^2 - 4)^2$

In Problems 37 and 38, find the val
$f(x)$. Then give the complete facto

37. 3; $f(x) = 2x^3 - 2x^2 + k$

In Problems 39 and 40, find a polyn
is given in the figure.

39. degree 3

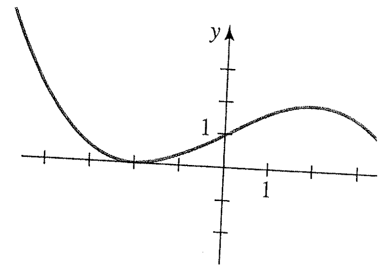


FIGURE 4.3.1 Graph for Problem 39

For Discussion

41. Discuss:

(a) For what positive integer valu

(b) For what positive integer valu

42. Suppose f is a polynomial functio
$f(x)$ have three complex zeros? Pu
cubic polynomial function be a rea

43. What is the smallest degree that a p
can have such that $1 + i$ is a comp

44. Let $z = a + bi$. Show that $z + \bar{z}$ a

45. Let $z = a + bi$. Use the results of

$f(x) = ($

is a polynomial function with real c

46. Try to prove or disprove the followi

In other words, $g(x) = 12f(x)$. If c is a zero of the function g, then c is also zero of f because $g(c) = 12f(c) = 0$ implies $f(c) = 0$. After working through the numbers in the list of potential rational zeros

$$\frac{p}{s}: \pm 1, \pm 3, \pm 9, \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \pm\tfrac{9}{2}, \pm\tfrac{1}{5}, \pm\tfrac{3}{5}, \pm\tfrac{9}{5}, \pm\tfrac{1}{10}, \pm\tfrac{3}{10}, \pm\tfrac{9}{10},$$

we find that $-\frac{1}{5}$ and $\frac{3}{2}$ are zeros of g, and hence are zeros of f. ≡

4.4 Exercises

Answers to selected odd-numbered problems begin on page ANS-13.

In Problems 1–20, find all rational zeros of the given polynomial function f.

Just list them (rational root thm)

1. $f(x) = 5x^3 - 3x^2 + 8x + 4$
2. $f(x) = 2x^3 + 3x^2 - x + 2$
3. $f(x) = x^3 - 8x - 3$
4. $f(x) = 2x^3 - 7x^2 - 17x + 10$
5. $f(x) = 4x^4 - 7x^2 + 5x - 1$
6. $f(x) = 8x^4 - 2x^3 + 15x^2 - 4x - 2$
7. $f(x) = x^4 + 2x^3 + 10x^2 + 14x + 21$
8. $f(x) = 3x^4 + 5x^2 + 1$
9. $f(x) = 6x^4 - 5x^3 - 2x^2 - 8x + 3$
10. $f(x) = x^4 + 2x^3 - 2x^2 - 6x - 3$
11. $f(x) = x^4 + 6x^3 - 7x$
12. $f(x) = x^5 - 2x^2 - 12x$
13. $f(x) = x^5 + x^4 - 5x^3 + x^2 - 6x$
14. $f(x) = 128x^6 - 2$
15. $f(x) = \frac{1}{2}x^3 - \frac{9}{4}x^2 + \frac{17}{4}x - 3$
16. $f(x) = 0.2x^3 - x + 0.8$
17. $f(x) = 2.5x^3 + x^2 + 0.6x + 0.1$
18. $f(x) = \frac{3}{4}x^3 + \frac{9}{4}x^2 + \frac{5}{3}x + \frac{1}{3}$
19. $f(x) = 6x^4 + 2x^3 - \frac{11}{6}x^2 - \frac{1}{3}x + \frac{1}{6}$
20. $f(x) = x^4 + \frac{5}{2}x^3 + \frac{3}{2}x^2 - \frac{1}{2}x - \frac{1}{2}$

Tricky: The constant coeff is zero!

In Problems 21–30, find all real zeros of the given polynomial function f. Then factor $f(x)$ using only real numbers.

21. $f(x) = 8x^3 + 5x^2 - 11x + 3$
22. $f(x) = 6x^3 + 23x^2 + 3x - 14$
23. $f(x) = 10x^4 - 33x^3 + 66x - 40$
24. $f(x) = x^4 - 2x^3 - 23x^2 + 24x + 144$
25. $f(x) = x^5 + 4x^4 - 6x^3 - 24x^2 + 5x + 20$
26. $f(x) = 18x^5 + 75x^4 + 47x^3 - 52x^2 - 11x + 3$
27. $f(x) = 4x^5 - 8x^4 - 24x^3 + 40x^2 - 12x$
28. $f(x) = 6x^5 + 11x^4 - 3x^3 - 2x^2$
29. $f(x) = 16x^5 - 24x^4 + 25x^3 + 39x^2 - 23x + 3$
30. $f(x) = x^6 - 12x^4 + 48x^2 - 64$

Finally, do this one by starting with checking $f(1)$

2. $y = 2$ is a horizontal asymptote for the graph of f

x	10	100	1000	10,000	100,000
$f(x)$					
x	−10	−100	−1000	−10,000	−100,000
$f(x)$					

In Problems 3–22, find the vertical and horizontal asymptotes for the graph of the given rational function. Find the x- and y-intercepts of the graph. Sketch the graph of f.

3. $f(x) = \dfrac{1}{x - 2}$

4. $f(x) = \dfrac{4}{x + 3}$

5. $f(x) = \dfrac{x}{x + 1}$

6. $f(x) = \dfrac{x}{2x - 5}$

7. $f(x) = \dfrac{4x - 9}{2x + 3}$

8. $f(x) = \dfrac{2x + 4}{x - 2}$

9. $f(x) = \dfrac{1 - x}{x + 1}$

10. $f(x) = \dfrac{2x - 3}{x}$

11. $f(x) = \dfrac{1}{(x - 1)^2}$

12. $f(x) = \dfrac{4}{(x + 2)^3}$

13. $f(x) = \dfrac{1}{x^3}$

14. $f(x) = \dfrac{8}{x^4}$

15. $f(x) = \dfrac{x}{x^2 - 1}$

16. $f(x) = \dfrac{x^2}{x^2 - 4}$

17. $f(x) = \dfrac{1}{x(x - 2)}$

18. $f(x) = \dfrac{1}{x^2 - 2x - 8}$

19. $f(x) = \dfrac{1 - x^2}{x^2}$

20. $f(x) = \dfrac{16}{x^2 + 4}$

21. $f(x) = \dfrac{-2x^2 + 8}{(x - 1)^2}$

22. $f(x) = \dfrac{x(x - 5)}{x^2 - 9}$

In Problems 23–30, find the vertical and slant asymptotes for the graph of the given rational function. Find the x- and y-intercepts of the graph. Sketch the graph of f.

23. $f(x) = \dfrac{x^2 - 9}{x}$

24. $f(x) = \dfrac{x^2 - 3x - 10}{x}$

25. $f(x) = \dfrac{x^2}{x + 2}$

26. $f(x) = \dfrac{x^2 - 2x}{x + 2}$

Graph these as we did on quiz (Dom, VA, HA, SA, X-int, y-int, Hole)

In Problems 35 and 36, find the]
a graphing utility to obtain the gra

35. $f(x) = \dfrac{x^3 - 3x^2 + 2x}{x^2 + 1}$

In Problems 37–40, find a ratio
no unique answer.

37. vertical asymptote: $x = 2$
horizontal asymptote: $y =$
x-intercept: $(5, 0)$

39. vertical asymptotes: $x = -$
horizontal asymptote: $y =$
x-intercept: $(3, 0)$

In Problems 41–44, find the asy
function. Find the x- and y-inte

41. $f(x) = \dfrac{x^2 - 1}{x - 1}$

43. $f(x) = \dfrac{x + 1}{x(x^2 + 4x + 3)}$

Miscellaneous Applications

45. **Parallel Resistors** A 5-oh
as shown in FIGURE 4.6.11. T
resistance r (in ohms) of th

Sketch the graph of R as a f
resistance R as r becomes v

46. **Power** The electrical powe

where E is the voltage of th
resistance in the circuit. Sk
$E = 5$ volts and $R = 1$ ohn

47. **Illumination Intensity** Th
point is directly proportiona
tional to the square of the di
strengths 16 units and 2 uni