

Page	1	2	3	4	5	Total	Course Points
Points	20	28	22	24	16	110	150
Score							

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam.

1. (6 points) Rewrite the expression so that it contains only positive exponents. Simplify your answer as much as possible.

$$\frac{15x^2(y^{-2}z)^3}{24x^7y^3z^{-6}} = \frac{5x^{2-7}y^{-6}z^3}{8y^3z^{-6}} = \frac{5x^{-5}y^{-6-3}z^{3-6}}{8y^3z^{-6}} = \boxed{\frac{5z^9}{8x^5y^9}}$$

2. (8 points) Simplify each expression completely:

$$\begin{aligned} \text{a) } (\sqrt{3} - \sqrt{12})^2 &= (\sqrt{3} - \sqrt{12})(\sqrt{3} - \sqrt{12}) = 3 - 2\sqrt{36} + 12 \\ &= 15 - 2(6) = \boxed{3} \end{aligned}$$

$$\text{b) } -3\sqrt{48x^5} + 7x\sqrt{75x^3}$$

$$\begin{aligned} &= -3\sqrt{16 \cdot 3 \cdot x^4 \cdot x} + 7x\sqrt{25 \cdot 3 \cdot x^2 \cdot x} \\ &= -3 \cdot 4 \cdot x^2 \sqrt{3x} + 7x \cdot 5 \cdot x \sqrt{3x} = -12x^2 \sqrt{3x} + 35x^2 \sqrt{3x} \\ &= \boxed{27x^2 \sqrt{3x}} \end{aligned}$$

3. (6 points) For the expression $\frac{\sqrt[3]{3}}{\sqrt{2+\sqrt{5}}}$,

a) rationalize the numerator.

$$\frac{\sqrt[3]{3} \cdot \sqrt[3]{3^2}}{(\sqrt{2+\sqrt{5}})\sqrt[3]{3^2}} = \boxed{\frac{3}{(\sqrt{2+\sqrt{5}})\sqrt[3]{3^2}}}$$

b) rationalize the denominator.

$$\frac{\sqrt[3]{3}}{\sqrt{2+\sqrt{5}}} \cdot \frac{\sqrt{2-\sqrt{5}}}{\sqrt{2-\sqrt{5}}} = \frac{\sqrt[3]{3}(\sqrt{2-\sqrt{5}})}{2-5} = \boxed{\frac{\sqrt[3]{3}(\sqrt{2-\sqrt{5}})}{-3}}$$

4. (6 points) Multiply and simplify (combine like terms):

$$(x^3 + 3x - 5)(x^2 - 9) = x^5 - 9x^3 + 3x^3 - 27x - 5x^2 + 45$$

$$= \boxed{x^5 - 6x^3 - 5x^2 - 27x + 45}$$

5. (6 points) Divide. Simplify your answer as much as possible. Specify any necessary domain restrictions.

$$\frac{2x^2 + 3x + 1}{x^2 + 2x - 15} \div \frac{x^2 + 6x + 5}{2x^2 - 7x + 3} = \frac{(2x+1)(x+1)}{(x-3)(x+5)} \cdot \frac{(2x-1)(x-3)}{(x+5)(x+1)}$$

$$= \boxed{\frac{(2x+1)(2x-1)}{(x+5)^2}, \quad x \neq 3, -1}$$

(x ≠ -5 is seen)

6. (8 points) Rewrite $5x^2 + 30x + 41$ as the sum of a perfect square and a constant by using the method of Completing the Square. Your answer should be in the form: $a(x-h)^2 + k$.

$$5(x^2 + 6x) + 41 = 5(x^2 + 6x + 3^2) + 41 - 5(3^2)$$

$$= \boxed{5(x+3)^2 - 4}$$

7. (8 points) Use long division to divide: $\frac{x^4 - 18x^2 + 5x + 2}{x^2 + 4x - 1}$

$$x^2 - 4x - 3 + \frac{21x + 5}{x^2 + 4x - 1}$$

$$x^2 + 4x + 1 \overline{) x^4 + 0x^3 - 18x^2 + 5x + 2}$$

$$- \underline{x^4 + 4x^3 + x^2}$$

$$-4x^3 - 19x^2 + 5x$$

$$- \underline{-4x^3 - 16x^2 - 4x}$$

$$-3x^2 + 9x + 2$$

$$- \underline{-3x^2 - 12x - 3}$$

$$21x + 5$$

6. (22 points) Solve each of the following equations for x .

a) $2\sqrt{x+1} - \sqrt{2x+3} = 1$

$$(2\sqrt{x+1})^2 = (\sqrt{2x+3} + 1)^2$$

$$4(x+1) = 1 + 2\sqrt{2x+3} + 2x+3$$

$$4x + 4 - 1 - 3 - 2x = 2\sqrt{2x+3}$$

$$2x = 2\sqrt{2x+3}$$

$$\rightarrow x = \sqrt{2x+3}$$

$$x^2 = 2x+3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

Check: Both work!

b) $x^2 - 17x + 52 = 0$

$$(x-4)(x-13) = 0$$

$$x = 4, 13$$

c) $3x^3 + 5x^2 - 12x - 20 = 0$

$$x^2(3x+5) - 4(3x+5) = 0$$

$$(x^2-4)(3x+5) = 0$$

$$(x+2)(x-2)(3x+5) = 0$$

$$x = 2, -2, -5/3$$

d) $\frac{6}{x^2-1} + \frac{3}{x+1} = \frac{x}{x-1}$ LCD is x^2-1

$$\frac{6 + 3(x-1)}{x^2-1} = \frac{x(x+1)}{x^2-1}$$

$$6 + 3x - 3 = x^2 + x$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x = 3, -1$$

Discard, as it is not in domain of given

7. (16 points) Suppose $f(x) = \frac{x+2}{5-x}$

a) Find the domain of f .

$x \neq 5$

b) Show that f is a one-to-one function, using the *definition* of one-to-one.

If $f(a) = f(b)$
then $a = b$.

Write: $f(a) = \frac{a+2}{5-a}$, $f(b) = \frac{b+2}{5-b}$

Assume: $\frac{a+2}{5-a} = \frac{b+2}{5-b}$

Now cross-multiply + cancel to get $a = b$

c) Find $f^{-1}(x)$, the inverse function of f .

① $y = \frac{x+2}{5-x}$

③ $(5-y)x = y+2$

② $x = \frac{y+2}{5-y}$

④ $5x - xy = y + 2$

⑤ $5x - 2 = y(x+1)$

d) Find the range of f .

$5x - 2 = xy + y$
factor out y

⑥ $y = \frac{5x-2}{x+1}$

⑦ $f^{-1}(x) = \frac{5x-2}{x+1}$

Range $f = \text{Dom } f^{-1}$
 $= \boxed{x \neq -1}$

8. (8 points) Suppose $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ -2x + 4 & \text{if } x \geq 0 \end{cases}$

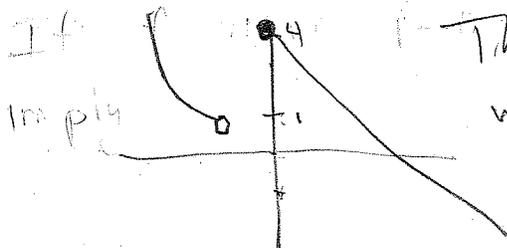
a) Find the domain of f . Express your answer using interval notation.

$\text{Dom } f: (-\infty, -1) \cup [0, \infty)$

b) Evaluate $f(-3) = (-3)^2 = \boxed{9}$

c) Evaluate $f(5) = -2(5) + 4 = \boxed{-6}$

d) Is f a one-to-one function? Give a *specific* reason why or why not.



There are infinite pairs of (x, y) where $f(x) = f(y)$ does not mean $x = y$. That is, it fails the horizontal line test.

9. (16 points) Suppose $f(x) = \frac{8}{x^2-4}$ and $g(x) = \sqrt{6-2x}$

a) Find the domain of f .

$$x \neq \pm 2$$

b) Find the domain of g .

$$6-2x \geq 0$$

$$x \leq 3$$

c) Find $(f \circ g)(x)$.

$$f(\sqrt{6-2x}) = \frac{8}{\sqrt{6-2x}^2 - 4} = \frac{8}{6-2x-4} = \frac{8}{2-2x} \text{ or } \frac{4}{1-x}$$

d) Find the domain of $f \circ g$.

$$x \leq 3 \text{ and } x \neq 1$$

e) Is f an even function, odd function, or neither? Justify your answer algebraically.

$f(x)$ is even since $f(x) = f(-x)$
that is, $\frac{8}{x^2-4} = \frac{8}{(-x)^2-4}$

f) Is g an even function, odd function, or neither? Justify your answer algebraically.

Neither, $g(x) \neq g(-x)$ so it's not even.

$$g(-x) = \sqrt{6-2(-x)} = \sqrt{6+2x} \text{ you}$$

$$-g(x) = -\sqrt{6-2x} \leftarrow \text{not equal}$$

