Math 220: Exam 3 Name My Key

Spring 2016 Instructor

Problem	1	2	3	4	5	6	Total	Course Points
Points	12	16	16	16	10	30	100	150
Score								

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- You may not use L'Hôpital's Rule to evaluate limits in this course.
- There are only problems on the front of the pages. Use the back of the pages for scrap paper.
- You will have exactly 60 minutes to complete the exam.

1. (12 points)
Given: $f(x, y) = ye^{2x} - y\ln y + 6x - 8$

$f_x = \boxed{2ye^{2x} + 6}$ $f_{xx} = \boxed{4ye^{2x}}$ $f_{xy} = \boxed{2e^{2x}}$

$f_y = e^{2x} - (1 \cdot \ln y + y \cdot \frac{1}{y})$

$= \boxed{e^{2x} - \ln y + 1}$

$f_{yy} = \boxed{-\frac{1}{y}}$ $f_{yx} = \boxed{2e^{2x}}$

2. (16 points) A manufacturer has an order for 1000 units that can be produced at two locations. Let x and y be the number of units produced at the two plants. Use the Method of Lagrange Multipliers to find the number of units that should be produced at each plant to minimize the cost if the cost function is given by:

$$C(x, y) = 0.25x^2 + 10x + 0.15y^2 + 12y + 5000$$

$$x + y = 1000 \longrightarrow \underbrace{x + y - 1000}_{g(x,y)} = 0$$

$$F(x, y, \lambda) = 0.25x^2 + 10x + 0.15y^2 + 12y + 5000 + \lambda(x + y - 1000)$$

$$\begin{cases} F_x = .5x + 10 + \lambda = 0 \\ F_y = .3y + 12 + \lambda = 0 \\ F_\lambda = x + y - 1000 = 0 \end{cases}$$

eliminate λ:

$$.5x + 10 = .3y + 12$$

solve for x in terms of y:

$$x = \frac{.3y + 2}{.5}$$

$$\boxed{x = \frac{3}{5}y + 4}$$

plug into F_λ to solve for y

$$\frac{3}{5}y + 4 + y - 1000 = 0$$

$$\frac{8}{5}y = 996 \longrightarrow y = \frac{(996)(5)}{(8)}$$

$$\boxed{y = 622.5}$$ plant B

find x:

$$x = 1000 - 622.5$$

$$\boxed{x = 377.5}$$ plant A

$$\boxed{x = 378}$$

$$\boxed{y = 622}$$

3. $f(x,y) = -x^3 + y^3 - 5x^2 + y^2 - 7x - y + 2$

Find all critical pts for f & determine their character. (Local max, min, saddle pt). Show your work clearly!!

Hint: $D\big|_{(x_0,y_0)} = f_{xx} f_{yy} - (f_{xy})^2$

$f_x = -3x^2 - 10x - 7 = -(3x^2 + 10x + 7) = -(3x+7)(x+1) = 0;$

$x = -7/3, -1$

$f_y = 3y^2 + 2y - 1 = (3y-1)(y+1) = 0;\ y = 1/3, -1$

The crit pts are $(-7/3, 1/3), (-7/3, -1), (-1, 1/3), (-1,-1)$

Notice each of these "satisfy" $f_x = 0$, $f_y = 0$, albeit trivially, since neither f_x nor f_y were multi-variable. you didn't need to plug x from $f_x = 0$ into $f_y = 0$ to get y. But in most of the examples, we did have to.

$f_{xx} = -6x - 10$, $f_{yy} = 6y + 2$, $f_{xy} = 0$

$(-7/3, 1/3)$: $D = (+14-10)(4) - 0^2 > 0$ & $f_{xx} > 0$, so $(-7/3, 1/3)$ is a local minimum

$(-7/3, -1)$: $D = (14-10)(-4) - 0^2 < 0$, so $(-7/3, -1)$ is a saddle pt

$(-1, 1/3)$: $D = (6-10)(4) - 0^2 < 0$, so $(-1, 1/3)$ is a saddle pt

$(-1,-1)$: $D = (6-10)(-4) - 0^2 > 0$, & $f_{xx} < 0$, so $(-1,-1)$ is a local maximum

Remember: $D > 0$ gives one or ~~the~~ other max, min.

The SDT as before points to concavity up (min)

down (max)

4. (16 points) Suppose the demand for a product is given by $q = \sqrt{1800 - 4p^2}$ (where p is the price in dollars and q is the quantity of items sold).

a) Find the elasticity function $E(p)$.

$$E(p) = -\frac{p}{q} \cdot \frac{dq}{dp} = \frac{-p}{(1800-4p^2)^{1/2}} \cdot \frac{1}{2}(1800-4p^2)^{-1/2} \cdot (-8p)$$

$$= \frac{8p^2}{2(1800-4p^2)} = \boxed{\frac{4p^2}{1800-4p^2}}$$

b) If the current price is \$20 per item, would a slight increase in price lead to an increase or decrease in revenue? Justify your answer using the elasticity function you found in part a).

$$E(20) = \frac{4(20)^2}{1800-4(20)^2} = \frac{1600}{200} = 8 > 1$$

The product is elastic with respect to price's effect on demand. A small increase in price above \$20 ~~\$40~~ will tend to decrease revenue, since

~~[illegible]~~

marginal revenue

$$\longrightarrow R'(p) = q(p)(1-E) < 0 \quad \text{when } E > 0$$

5. (10 points)

Suppose $g(x) = x^4 - 8x^3 + 16x^2 + 3$

Find the absolute maximum and the absolute minimum of g on the interval [-1, 3]. Show all work.

$g'(x) = 4x^3 - 24x^2 + 32x = 4x(x^2 - 6x + 8) = 0$

$4x = 0$ $\qquad$ $x^2 - 6x + 8 = 0$

$x = 0$ $\qquad$ $(x-4)(x-2) = 0$

critical values $\qquad$ $x = 4, 2$ $\Longrightarrow$ Only $x = 0$ & 2 are in $[-1, 3]$

$g''(x) = 12x^2 - 48x + 32$

$g''(0) = 32 > 0$, so $g(x)$ is concave up at $x = 0$, and $g(0)$ is therefore a local min.

$g''(2) = -16 < 0$, so $g(x)$ is concave down at $x = 2$ and therefore $g(0)$ is a local max.

Compare g at endpts of $[-1, 3]$ to $g(0)$ and $g(2)$

$g(-1) = 28$, $\boxed{g(0) = 3}$, $g(2) = 19$, $\boxed{g(3) = 112}$

abs. min $\qquad$ abs max

6. (30 points) Suppose $f(x) = \frac{2x^2}{x^2-1}$

Then $f'(x) = \frac{-4x}{(x^2-1)^2}$ and $f''(x) = \frac{12x^2+4}{(x^2-1)^3}$

a) Find the domain of f.

$x^2 - 1 \neq 0$ so $x \neq \pm 1$

Important: There are V.A. at $x = 1$ & $x = -1$

b) Find the x and y intercepts on the graph of f.

$f(0) = \frac{2(0^2)}{0^2 - 1} = 0 \rightarrow$ (0,0) is only intercept!

c) Indicate the intervals where f satisfies the given condition. Support your answers in the space below.

f is increasing on $(-\infty, -1) \cup (-1, 0)$ i.e., where is $f' > 0$

f is decreasing on $(0, 1) \cup (1, \infty)$ i.e., where is $f' < 0$

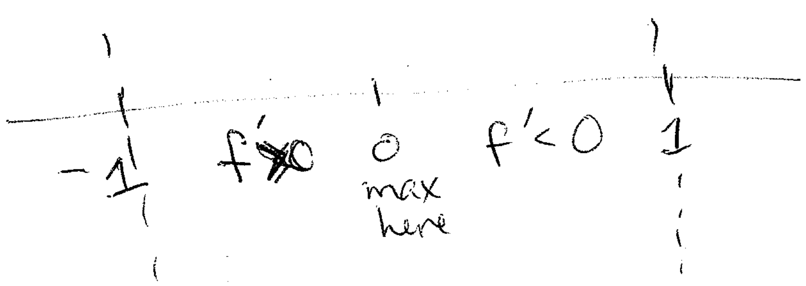
f is concave upward on $(-\infty, -1) \cup (1, \infty)$ i.e., where is $f'' > 0$

f is concave downward on $(-1, 1)$ i.e., where is $f'' < 0$

Here's why:

$f'(x) = \frac{-4x}{(x^2-1)^2} = 0$ at $x = 0$, which is the only critical point.

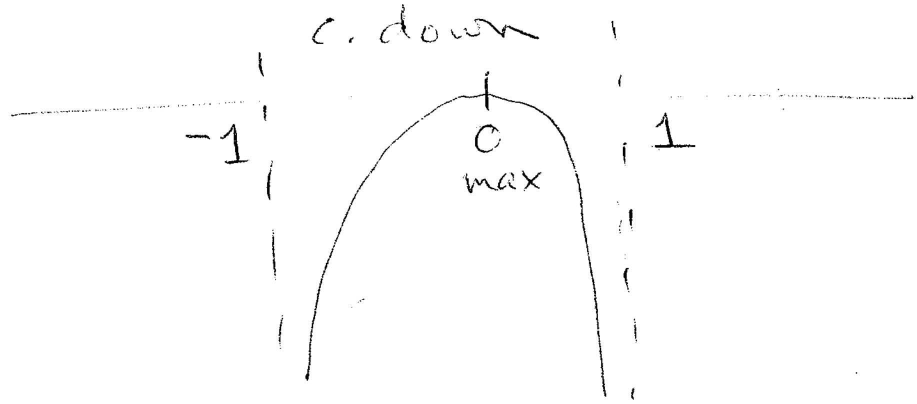
$f''(0) = \frac{12 \cdot 0^2 + 4}{(0^2 - 1)^3} = \frac{4}{-1} = -4$, less than zero, so $f(0)$ is a local max (concave down).



We don't need to test either side of zero. But ... (over) $\rightarrow$

We need to test left of $x=-1$ and right of $x=1$.

So far:



Check $x=-2$

& $x=2$

by substituting into f' for increasing / decreasing ;

f'' for concave up/down

FDT: $f'(-2) = \frac{8}{+} > 0$, $f \uparrow$ on $(-\infty, -1)$

$f'(2) = \frac{-8}{+} < 0$, $f \downarrow$ on $(1, \infty)$

Also, from $f(0)$ local max, $f \uparrow$ on $(-1, 0)$

and $f \downarrow$ on $(0, 1)$.

SDT: f c. down on $(-1, 1)$ from local max

$f''(-2) = \frac{+}{+} > 0$, f c. up on $(-\infty, -1)$

$f''(2) = \frac{+}{+} > 0$, f c. up on $(1, \infty)$

d) Find ordered pairs for each of the following. Write "none" if the function does not have the specified characteristic. Provide support for your answers in the space below.

f has a local minimum at nowhere

f has a local maximum at $(0,0)$

f has points of inflection at nowhere

see critical value of $x = 0$ & $f''(0) < 0$ there (c. down)

No POI because concavity does not change anywhere other than either side of vertical asymptotes. These are not in the domain.

e) Give the equations of all asymptotes. Write "none" if the function does not have any asymptotes of the specified type. Use appropriate limits to justify your answer in the space below.

f has horizontal asymptote(s) $y = 1$ because deg P = deg Q (ratio of lead coeffs)

f has vertical asymptote(s) $x = 1$, $x = -1$

$\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$

$\lim_{x \to 1^-} f(x) = -\infty$, $\lim_{x \to 1^+} f(x) = \infty$

Via sign analysis — but concavity + intercepts + max give us this info.

f) On the next page, use the information found in parts a) through e) to sketch the graph of f. Be sure to label your axes appropriately.

GRAPH FOR PROBLEM #6