

Math 220: Exam 1

Name My Key

Spring 2016

Instructor _____

Page	1	2	3	4	Total	Course Points
Points	16	30	26	28	100	150
Score						

- Calculators are not permitted for this test.
- Show your work unless the problem requires only a short answer.
- There are problems on both front and back of the pages.
- If you need scrap paper, ask your instructor. You may not use your own. If you do use scrap paper, make sure to hand it in at the end of the exam.

1. (16 points) Find the domain of each function.

a) $f(x) = \frac{x^2-1}{x^2-2x-3}$

$$x^2 - 2x - 3 = (x-3)(x+1)$$

$$x \neq 3, -1$$

b) $h(x) = \ln(2x-1)$

$$2x-1 > 0,$$

$$x > \frac{1}{2}$$

c) $j(x) = \sqrt{x^2-9}$

$$x^2 - 9 \geq 0$$

$$x \geq 3 \text{ or } x \leq -3$$

d) $k(x) = \frac{\sqrt[3]{x+4}}{3^x-1}$

$$3^x - 1 \neq 0, \quad 3^x \neq 1,$$

$$x \neq 0$$

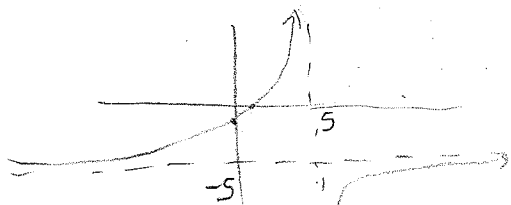
$$x+4 \in \mathbb{R}, \quad x \in \mathbb{R}$$

2. (30 points) Evaluate the indicated limits. Show your work, where applicable.

a) $\lim_{x \rightarrow 5^-} \frac{-5x+2}{x-5}$

x	f(x)
4.5	$\frac{-22.5}{-0.5} = +$
4.8	$\frac{-23.5}{-0.2} = +$

↓ gets larger



b) $\lim_{x \rightarrow 7} \frac{x^2-2x-35}{2x-14}$

$$= \lim_{x \rightarrow 7} \frac{(x-7)(x+5)}{2(x-7)} = \boxed{6}$$

c) $\lim_{x \rightarrow 2} \frac{\frac{1}{2} - \frac{1}{x}}{x^2-4}$

$$= \lim_{x \rightarrow 2} \frac{\frac{x-2}{2x}}{(x-2)(x+2)} = \lim_{x \rightarrow 2} \frac{1}{2x(x+2)} = \boxed{\frac{1}{16}}$$

d) $\lim_{x \rightarrow 4} \frac{\sqrt{x+12}}{\sqrt{x}+3}$

$$= \boxed{\frac{4}{5}}$$

e) Given $f(x) = \begin{cases} x+3 & \text{if } x < 5 \\ 6 & \text{if } x = 5 \\ x-3 & \text{if } x > 5 \end{cases}$

$\lim_{x \rightarrow 5^-} f(x) =$ 8

$\lim_{x \rightarrow 5^+} f(x) =$ 2

$\lim_{x \rightarrow 5} f(x) =$ DNE

3. (12 points) Determine whether the function below is continuous or not at the indicated values of x . Give a thorough explanation why or why not, using the definition of continuity.

$$f(x) = \begin{cases} 2x + 3 & \text{if } x < -4 \\ x^2 + 4x + 3 & \text{if } -4 \leq x < 1 \\ 3x^2 + 5 & \text{if } x > 1 \end{cases}$$

LHL as $x \rightarrow -4$ is -5
RHL as $x \rightarrow -4$ is 3

a) at $x = -4$

No, b/c LHL \neq RHL
Or, $f(-4) = \text{RHL only.}$

b) at $x = 0$

Yes, b/c $f(0) = 3$ and $\lim_{x \rightarrow 0} f(x) = 3$

c) at $x = 1$

Yes, $f(1)$ DNE but

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = 8 = \lim_{x \rightarrow 1^+} f(x) = 8 = \text{RHL}$$

4. (14 points) Suppose $f(x) = \frac{2}{x}$.

a) Use the definition of derivative to show that the derivative function for f is $f'(x) = -\frac{2}{x^2}$. You must use the definition. No credit will be given for any other method.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{2x - 2x - 2h}{(x+h)x}}{h} = \lim_{h \rightarrow 0} \frac{-2h}{h(x+h)x} = \frac{-2}{x^2} \end{aligned}$$

b) Find the equation of the line tangent to the graph of f at $x = -3$. Any form of linear equation is acceptable.

$$y - f(-3) = \frac{-2}{(-3)^2} (x - -3)$$

$$\boxed{y + \frac{2}{3} = -\frac{2}{9}(x + 3)}$$

5. (12 points) Suppose you invest \$400 at an annual interest rate of 4% for 25 years.

a) How much money will you have at the end of 25 years if the interest is compounded monthly?
You do not need to simplify your answer.

$$P(25) = 400 \left(1 + \frac{.04}{12} \right)^{(12)(25)}$$

b) If your interest is compounded continuously, will your investment reach \$1000 by the end of the 25 years? Justify your answer.

$$400 e^{(.04)(25)} \stackrel{?}{>} 1000$$

$$e^{1} \stackrel{?}{>} \frac{10}{4} = 2.5 \quad \boxed{\text{yes}}$$

6. (16 points) Dylan owns an apartment complex that consists of 25 apartment units. The monthly cost to maintain the complex is \$3000 plus \$80 for each of the units that is rented. Dylan charges his tenants \$800/month for rent.

a) Write a function $C(x)$ that describes Dylan's monthly cost as a function of the number of units that he has rented.

$$C(x) = 80x + 3000$$

b) How many apartments must Dylan rent so that he does not lose money?

$$R(x) = 800x \quad ; \quad R(x) = C(x) \quad \text{when} \quad 720x = 3000$$

$$C(x) = 80x + 3000 \quad \boxed{x=5} \quad x = \frac{3000}{720}$$

$$4Lx < 5$$

c) Last month, all 25 of Dylan's apartments were rented. How much PROFIT did Dylan make last month?

$$P = R - C \Rightarrow P(25) = 800(25) - 80(25) - 3000$$

$$= 20,000 - 2000 - 3000 = \boxed{\$15,000}$$

d) This month, Dylan's apartments have bedbugs. Five tenants have moved out. The remaining twenty tenants insist that they will stay only if Dylan reduces the rent significantly. How low can Dylan set the rent and still make a profit with only twenty tenants?

$$P(20) = 20p > 80(20) + 3000$$

$$P(20) \Rightarrow p > \frac{4600}{20} = \$230$$