Functions - Logarithmic Functions

Objective: Convert between logarithms and exponents and use that relationship to solve basic logarithmic equations.

The inverse of an exponential function is a new function known as a logarithm. Lograithms are studied in detail in advanced algebra, here we will take an introductory look at how logarithms works. When working with radicals we found that there were two ways to write radicals. The expression $\sqrt[m]{a^n}$ could be written as $a^{\frac{n}{m}}$. Each form has its advantages, thus we need to be comfortable using both the radical form and the rational exponent form. Similarly an exponent can be written in two forms, each with its own advantages. The first form we are very familiar with, $b^x = a$, where b is the base, a can be thought of as our answer, and x is the exponent. The second way to write this is with a logarithm, $\log_b a = x$. The word "log" tells us that we are in this new form. The variables all still mean the same thing. b is still the base, a can still be thought of as our answer.

Using this idea the problem $5^2 = 25$ could also be written as $\log_5 25 = 2$. Both mean the same thing, both are still the same exponent problem, but just as roots can be written in radical form or rational exponent form, both our forms have their own advantages. The most important thing to be comfortable doing with logarithms and exponents is to be able to switch back and forth between the two forms. This is what is shown in the next few examples.

Example 535.

Write each exponential equation in logarithmic form

$m^3 = 5$ Identify base, m, answer, 5, and exponent 3

$\log_m 5 = 3$ Our Solution

$7^2 = b$ Identify base, 7, answer, b, and exponent, 2

$\log_7 b = 2$ Our Solution

$\left(\frac{2}{3}\right)^4 = \frac{16}{81}$ Identify base, $\frac{2}{3}$, answer, $\frac{16}{81}$, and exponent 4

$\log_{\frac{2}{3}} \frac{16}{81} = 4$ Our Solution

Example 536.

Write each logarithmic equation in exponential form

$\log_4 16 = 2$ Identify base, 4, answer, 16, and exponent, 2

$4^2 = 16$ Our Solution

$$\begin{aligned} \log_3 x &= 7 && \text{Identify base, 3, answer, } x\text{, and exponent, 7} \\ 3^7 &= x && \text{Our Solution} \end{aligned}$$

$$\begin{aligned} \log_9 3 &= \frac{1}{2} && \text{Identify base, 9, answer, 3, and exponent, } \frac{1}{2} \\ 9^{\frac{1}{2}} &= 3 && \text{Our Solution} \end{aligned}$$

Once we are comfortable switching between logarithmic and exponential form we are able to evaluate and solve logarithmic expressions and equations. We will first evaluate logarithmic expressions. An easy way to evaluate a logarithm is to set the logarithm equal to x and change it into an exponential equation.

Example 537.

$$\begin{aligned} \text{Evaluate } \log_2 64 & && \text{Set logarithm equal to } x \\ \log_2 64 &= x && \text{Change to exponent form} \\ 2^x &= 64 && \text{Write as common base, } 64 = 2^6 \\ 2^x &= 2^6 && \text{Same base, set exponents equal} \\ x &= 6 && \text{Our Solution} \end{aligned}$$

$x = 6$ Our Solution

Example 538.

$$\begin{aligned} \text{Evaluate } \log_{125} 5 & && \text{Set logarithm equal to } x \\ \log_{125} 5 &= x && \text{Change to exponent form} \\ 125^x &= 5 && \text{Write as common base, } 125 = 5^3 \\ (5^3)^x &= 5 && \text{Multiply exponents} \\ 5^{3x} &= 5 && \text{Same base, set exponents equal } (5 = 5^1) \\ 3x &= 1 && \text{Solve} \\ \overline{3} \ \ & \ \overline{3} && \text{Divide both sides by 3} \\ x &= \frac{1}{3} && \text{Our Solution} \end{aligned}$$

Example 539.

$$\begin{aligned} \text{Evaluate } \log_3 \frac{1}{27} & && \text{Set logarithm equal to } x \\ \log_3 \frac{1}{27} &= x && \text{Change to exponent form} \\ 3^x &= \frac{1}{27} && \text{Write as common base, } \frac{1}{27} = 3^{-3} \\ 3^x &= 3^{-3} && \text{Same base, set exponents equal} \\ x &= -3 && \text{Our Solution} \end{aligned}$$

World View Note: Dutch mathematician Adriaan Vlacq published a text in 1628 which listed logarithms calculated out from 1 to 100,000!

Solve equations with logarithms is done in a very similar way, we simply will change the equation into exponential form and try to solve the resulting equation.

Example 540.

$$\begin{aligned} \log_5 x &= 2 && \text{Change to exponential form} \\ 5^2 &= x && \text{Evaluate exponent} \\ 25 &= x && \text{Our Solution} \end{aligned}$$

Example 541.

$$\begin{aligned} \log_2(3x+5) &= 4 && \text{Change to exponential form} \\ 2^4 &= 3x+5 && \text{Evaluate exponent} \\ 16 &= 3x+5 && \text{Solve} \\ \underline{-5 \qquad} & \underline{\quad -5} && \text{Subtract 5 from both sides} \\ 11 &= 3x && \text{Divide both sides by 3} \\ \overline{3} & \quad \overline{3} && \\ \frac{11}{3} &= x && \text{Our Solution} \end{aligned}$$

Example 542.

$$\begin{aligned} \log_x 8 &= 3 && \text{Change to exponential form} \\ x^3 &= 8 && \text{Cube root of both sides} \\ x &= 2 && \text{Our Solution} \end{aligned}$$

There is one base on a logarithm that gets used more often than any other base, base 10. Similar to square roots not writting the common index of 2 in the radical, we don't write the common base of 10 in the logarithm. So if we are working on a problem with no base written we will always assume that base is base 10.

Example 543.

$$\begin{aligned} \log x &= -2 && \text{Rewrite as exponent, 10 is base} \\ 10^{-2} &= x && \text{Evaluate, remember negative exponent is fraction} \\ \frac{1}{100} &= x && \text{Our Solution} \end{aligned}$$

This lesson has introduced the idea of logarithms, changing between logs and exponents, evaluating logarithms, and solving basic logarithmic equations. In an advanced algebra course logarithms will be studied in much greater detail.

10.5 Practice - Logarithmic Functions

Rewrite each equation in exponential form.

1) $\log_9 81 = 2$

2) $\log_b a = -16$

3) $\log_7 \frac{1}{49} = -2$

4) $\log_{16} 256 = 2$

5) $\log_{13} 169 = 2$

6) $\log_{11} 1 = 0$

Rewrite each equations in logarithmic form.

7) $8^0 = 1$

8) $17^{-2} = \frac{1}{289}$

9) $15^2 = 225$

10) $144^{\frac{1}{2}} = 12$

11) $64^{\frac{1}{6}} = 2$

12) $19^2 = 361$

Evaluate each expression.

13) $\log_{125} 5$

14) $\log_5 125$

15) $\log_{343} \frac{1}{7}$

16) $\log_7 1$

17) $\log_4 16$

18) $\log_4 \frac{1}{64}$

19) $\log_6 36$

20) $\log_{36} 6$

21) $\log_2 64$

22) $\log_3 243$

Solve each equation.

23) $\log_5 x = 1$

24) $\log_8 k = 3$

25) $\log_2 x = -2$

26) $\log n = 3$

27) $\log_{11} k = 2$

28) $\log_4 p = 4$

29) $\log_9 (n + 9) = 4$

30) $\log_{11} (x - 4) = -1$

31) $\log_5 (-3m) = 3$

32) $\log_2 -8r = 1$

33) $\log_{11} (x + 5) = -1$

34) $\log_7 -3n = 4$

35) $\log_4 (6b + 4) = 0$

36) $\log_{11} (10v + 1) = -1$

37) $\log_5 (-10x + 4) = 4$

38) $\log_9 (7 - 6x) = -2$

39) $\log_2 (10 - 5a) = 3$

40) $\log_8 (3k - 1) = 1$