Examples from Section 4:

Ex: Solve for x: $5^x = 6^{x-1}$

→ maybe use $\log_5(\quad)$, $\log_6(\quad)$

1st method: Try $\log_5(\quad)$ first.

$$\log_5(5^x) = \log_5(6^{x-1})$$

$$x\log_5(5) = (x-1)\log_5(6)$$

$$x(1) = (x-1)\log_5(6)$$

$$x = \log_5(6)\cdot x - \log_5(6)$$

$$(1-\log_5(6))x = -\log_5(6)$$

$$x = \frac{-\log_5(6)}{1-\log_5(6)}$$

2nd method: We can use a different log function.

Try $\log_6(\quad)$:

$$\log_6(5^x) = \log_6(6^{x-1})$$

$$x\log_6(5) = (x-1)\log_6(6)$$

$$x\log_6(5) = x-1$$

$$x\log_6(5) - x = -1$$

$$x = \frac{-1}{\log_6(5)-1}$$

→ These answers look different, but they're equal!

Ex: Solve for x:

$$\log_5(x+2) = 2$$

Method: Apply exponential, $5^{(\ )}$, to both sides.

$$5^{\log_5(x+2)} = 5^2$$

$$(x+2)\log_5(5) = 25$$

$$x+2 = 25 \longrightarrow \boxed{x = 23}$$

Let's look at more examples involving exponentials/logs.

Ex: Solve for x (from Section 4 probs, #20)

(a) $2^{x-3} = 32$

involves $2^{\square}$, so use $\log_2(\quad)$.
Though, any log base will do.

$$\rightarrow \log_2(2^{x-3}) = \log_2(32)$$

$$(x-3)\log_2(2) = \log_2(2^5)$$

$$x-3 = 5\log_2(2)$$

$$x-3 = 5 \longrightarrow \boxed{x = 8}$$

Alternatively, we could use the log def^{n}!

(e) $4^x = 3^{2x-1}$ (must use common log)

$$\log(4^x) = \log(3^{2x-1})$$

$$x\log(4) = (2x-1)\log(3)$$

$$x\log(4) = 2\log(3)x - \log(3)$$

$$x\log(4) - 2\log(3)x = -\log(3)$$

$$x(\log(4) - 2\log(3)) = -\log(3)$$

$$\boxed{x = \frac{-\log(3)}{\log(4) - 2\log(3)}}$$

optionally, $\log(4) - 2\log(3) = \log(4) - \log(9)$
$= \log(4/9)$

(g) $7(4^{6x-2}) + 13 = 41$

$$\rightarrow 7(4^{6x-2}) = 28$$

$$4^{6x-2} = 4$$

also: $4^b = 4^a \Rightarrow b = a$

Again, it's natural to use $\log_4(\quad)$, but any log base is fine.

$$\log_4(4^{6x-2}) = \log_4(4)$$

$$(6x-2)\log_4(4) = 1$$

$$6x - 2 = 1$$

$$\boxed{x = 1/2}$$

Be sure to practice the problems!

Section 5: Compound Interest.

Suppose you have P dollars to invest (here we use P for "principal," not "profit") that you deposit in a bank. The bank offers a 3% interest rate, compounded annually.

Def^n: (Present Value, P)

The amount of money at the beginning of an investment.

Def^n: (Future Value, F)

The amount of money at the end of an investment.

Interest rate = annual interest rate

Compounding = # of times interest is calculated in a year

How much money after 1 year?

$$\underbrace{P}_{\text{starting amount}} + \underbrace{.03P}_{3\% \text{ of starting amount}} = P(1+.03)$$

What about compounding monthly?

- **After 1 month:**

$$P + \left(\frac{.03}{\underline{12}}\right)P = P\left(1+\frac{.03}{12}\right)$$

new amt after 1 month

i.e. statement on Feb 1st

the 3% is for 1 year! So the interest earned over this month is 1/12th of that rate.

• After 2 months:

$$\left(\underline{\text{our new}}\ \underline{\text{starting amt.}}\right) + \overbrace{\left(\frac{.03}{12}\right)\left(\underline{\text{our new}}\ \underline{\text{starting amt.}}\right)}^{\text{interest on new starting amt}}$$

amt after 1^{st} month

$$= P\left(1+\frac{.03}{12}\right) + \left(\frac{.03}{12}\right) P\left(1+\frac{.03}{12}\right)$$

$$= P\left(1+\frac{.03}{12}\right)\left[1 + \frac{.03}{12}\right]$$

$$= P\left(1+\frac{.03}{12}\right)^2$$

• After 3 months:

$$P\left(1+\frac{.03}{12}\right)^2 + \left(\frac{.03}{12}\right) * P\left(1+\frac{.03}{12}\right)^2$$

$$= P\left(1+\frac{.03}{12}\right)^2\left[1 + \frac{.03}{12}\right] = P\left(1+\frac{.03}{12}\right)^3$$

• After 28 months:

$$P\left(1+\frac{.03}{12}\right)^{28}$$

What if we compounded interest weekly?

(1 yr = 52 wks)

(1 month ≈ 4 wks)

• **After 1 week:** $P + \left(\frac{.03}{52}\right)P$

• **After 1 month:** $P\left(1+\frac{.03}{52}\right)^4$

# of times you compound over 1 month.

• **After 3 months:** $P\left(1+\frac{.03}{12}\right)^{12}$

Ex: You have \$2000 in a bank w/ 5% compounded quarterly. How much money will we have after 7 years?

1st quarter:

$$2000 + \left(\frac{.05}{4}\right)2000 = 2000\left(1+\frac{.05}{4}\right)$$

1st year:

$$2000\left(1+\frac{.05}{4}\right)^4$$

7th year:

$$2000\left(1+\frac{.05}{4}\right)^{28} \approx 2831.98$$

Compounding interest (n-times in year)

$$\boxed{F = P\left(1+\frac{r}{n}\right)^{nt}}$$

alternatively, this is also the total # of compoundings that have passed.

F = future val , r = annual interest rate

P = present val , t = number of years of investment