Quiz 1 on Friday (Sept. 3rd at start of class)

Topics: • Section 3 (cost, revenue, profit)

• Algebra properties.

Ex: **Profit-and-loss analysis.** Boxowitz, Inc., a computer firm, is planning to sell a new graphing calculator. For the first year, the fixed costs for setting up the new production line are \$100,000. The variable costs for producing each calculator are estimated at \$20. The sales department projects that 150,000 calculators can be sold during the first year at a price of \$45 each.

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a) Find and graph $C(x)$, the total cost of producing x calculators.

b) Using the same axes as in part (a), find and graph $R(x)$, the total revenue from the sale of x calculators.

c) Using the same axes as in part (a), find and graph $P(x)$, the total profit from the production and sale of x calculators.

d) What profit or loss will the firm realize if the expected sale of 150,000 calculators occurs?

e) How many calculators must the firm sell in order to break even?

\# of calculators

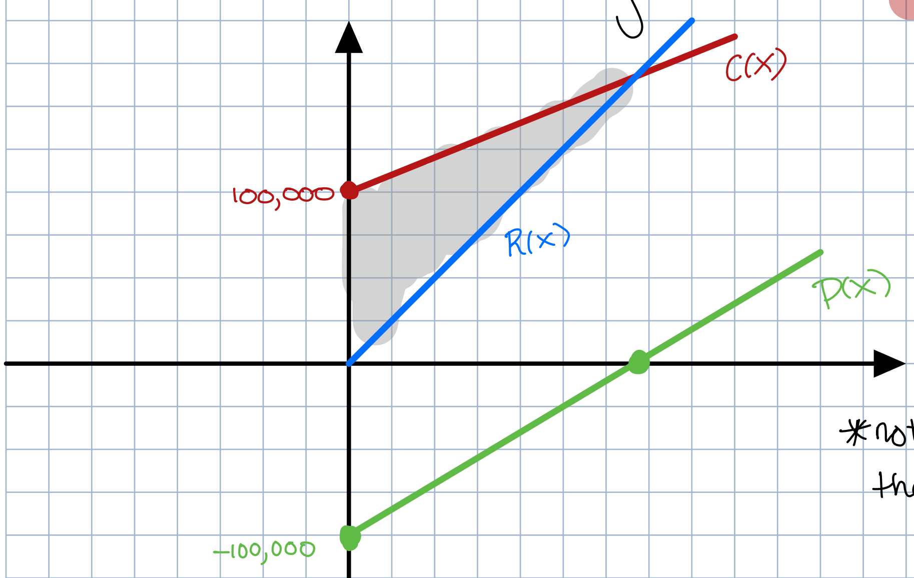
(a) $C(x) \rightarrow$ indicates that cost should be w.r.t x!

Fixed Costs: \$ 100,000

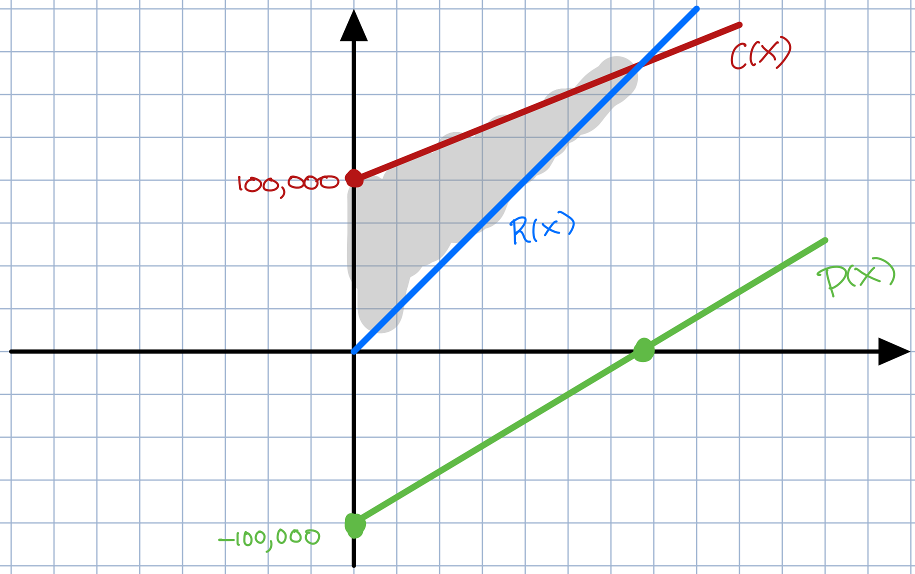
Variable costs: \$ 20 / calculator

var. cost should always be "per..."

These are the only costs $\Rightarrow$ $C(x) = 20x + 100000$



*not to scale cause that'd be unfeasible...



(b) $R(x) \longrightarrow$ indicates that revenue should be in terms of x also.

$p = \$45/\text{calculator}$

price

$$R(x) = 45x$$

(c) $P(x) \longrightarrow$ indicates that . . .

Profit = Revenue − Cost

gotta know!

$$P(x) = R(x) - C(x) = 45x - (20x + 100{,}000)$$

$$P(x) = 25x - 100{,}000$$

(d) Profit when $x = 150{,}000$ calculators

$$P(150000) = 25(150000) - 100{,}000$$

$$= 3{,}750{,}000 - 100{,}000$$

$$= \$3{,}650{,}000$$

also compare with conclusion in part (e)

(e) Q means "What x-value makes us break even?"

$\hookrightarrow$ Find $P(x) = 0$!

$$0 = 25x - 100000$$

$$x = \frac{100000}{25} = 4{,}000 \text{ calculators}$$

Ex:

(Bittinger)

Profit-and-loss analysis. Jimmy decides to mow lawns to earn money. The initial cost of his lawnmower is \$250. Gasoline and maintenance costs are \$4 per lawn.

a) Formulate a function $C(x)$ for the total cost of mowing x lawns.

b) Jimmy determines that the total-profit function for the lawnmowing business is given by $P(x) = 9x - 250$. Find a function for the total revenue from mowing x lawns. How much does Jimmy charge per lawn?

c) How many lawns must Jimmy mow before he begins making a profit?

(a) $C(x) \longrightarrow$ Jimmy's cost of mowing x lawns

Fixed: \$ 250

Variable: \$ 4/lawn

$$C(x) = 4x + 250$$

(b) $P(x) = 9x - 250$ ← Profit function

Profit = Revenue − Cost

$$\Longrightarrow R(x) = P(x) + C(x)$$

$$= 9x - 250 + (4x + 250)$$

$$R(x) = 13x$$

So in order to achieve Jimmy's projected Profit, he's gotta charge \$13/lawn

"marginal revenue" = "price"

Remark: this is all based on the <u>projected</u> profit function Jimmy came up with in part b.

(c) "Makes profit" $\approx P(x) > 0$

$\rightarrow$ So, first we'll find the x-value that makes $P(x)=0$.

$$9x - 250 = 0$$

$$\Rightarrow x = \frac{250}{9} = 27.\overline{7}$$

- 27 lawns is not enough to break even
- 28 will turn a profit

Ex: Simplify the logarithm expression

- $\log_4(16) = 2$, $4^2 = 16$
- $\log_5(1/125) = -3$
- $\log_2(16) = 4$, $2^4 = 16$
- $\log_{1/3}(81) = -4$
- $\log_3(27) = 3$, $3^3 = 27$
- $\log_{3/2}(27/8) = 3$

$$\boxed{\log_b(x) = y \iff b^y = x.}$$

Common Bases:

- Base $b=10$: "common log"

$$\boxed{\log_{10}(x) =: \log(x)}$$

no # means that base = 10

- Base $b = e$: "natural log"

$\boxed{e \approx 2.7182818\ldots}$ $\boxed{\log_e(x) =: \ln(x)}$

Ex: Simplify these logarithms:

- $\log(1000) \rightarrow \log_{10}(1000) = \log_{10}(10^3) = 3\log_{10}(10) = 3$
- $\log(1/100) \rightarrow \log_{10}(10^{-2}) = -2\log_{10}(10) = -2$
- $\ln(1/e) \rightarrow \log_e(e^{-1}) = (-1)\log_e(e) = -1$
- $\ln(\sqrt{e}) \rightarrow 1/2$
- $\log_{34}(34) \rightarrow 1$
- $\log_8(1) \rightarrow 0$

Ex: Write the following as a single logarithm.

(a) $\log_3(x+2y) - \log_3(x-y) = \log_3\left(\frac{x+2y}{x-y}\right)$

(b) $\log(x^2) + \frac{1}{2}\log(y) - \log(z)$

$= \log(x^2) + \log(\sqrt{y}) - \log(z)$

$= \log(x^2\sqrt{y}) - \log(z) = \log\left(\frac{x^2\sqrt{y}}{z}\right)$

(c) $\frac{1}{3}\left(\log_5(x) - 2\log_5(y)\right) + 5\log_5(z)$

$\rightarrow \frac{1}{3}\log_5(x) - \frac{2}{3}\log_5(y) + 5\log_5(z)$

$= \log_5(\sqrt[3]{x}) - \log_5(\sqrt[3]{y}^{\,2}) + \log_5(z^5)$

$= \log_5\left(\frac{\sqrt[3]{x}}{\sqrt[3]{y}^{\,2}}\right) + \log_5(z^5)$

$= \log_5\left(\frac{\sqrt[3]{x}\;z^5}{\sqrt[3]{y}^{\,2}}\right) = \log_5\left(\frac{x^{1/3}\;z^5}{y^{2/3}}\right)$

Change of Base:

How can we change from $\log_b(x)$ to log with a diff. base?

$$\boxed{\log_a(x) = \frac{\log_b(x)}{\log_b(a)}}$$

$a>0$
$a \neq 1$

Ex: Solve for x:

(a) $2(1+4^x) = 6$

$\rightarrow 1+4^x = 3 \rightarrow 4^x = 2$

$$\to (2^2)^x = 2$$

$$\to 2^{2x} = 2 \implies x = 1/2$$

(b) $2(1+4^x) = 12$

$$\to 1+4^x = 6 \to 4^x = 5$$

use $\log_4(x)$ $\implies$ $\log_4(4^x) = \log_4(5)$

$$x \log_4(4) = \log_4(5)$$

$$x = \log_4(5)$$

We can actually use any base log.

$\boxed{b=10}$

$$2(1+4^x) = 12$$

$$\to 4^x = 5$$

$$\log(4^x) = \log(5)$$

$$x \log(4) = \log(5)$$

$$x = \frac{\log(5)}{\log(4)}$$