Ex: Simplify

(a) $\left(\frac{2}{5}\right)^{-1} \longrightarrow \left(\frac{2}{5}\right)^{-1} = \frac{1}{(2/5)^1} = 5/2$

(b) $8^{2/3} \longrightarrow \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

(c) $\sqrt[4]{\sqrt[3]{x}} \longrightarrow \left((x)^{1/3}\right)^{1/4} = x^{(1/3 * 1/4)}$

$= x^{1/12}$

$= \sqrt[12]{x}$

(d) $\frac{x^2 y^{-5} z}{x^{-4} y z^3}$

$\longrightarrow$ x's: $\frac{x^2}{x^{-4}} = x^{2-(-4)} = x^6$

y's: $\frac{y^{-5}}{y} = y^{-5-1} = y^{-6}$

z's: $\frac{z}{z^3} = z^{1-3} = z^{-2}$

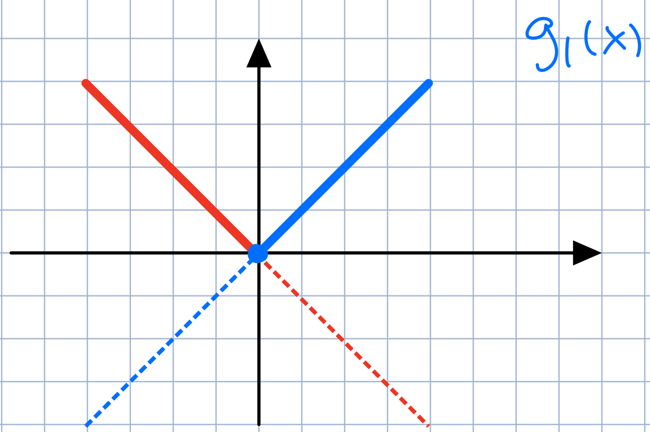
So, our expression simplifies to: $x^6 y^{-6} z^{-2}$

$= \frac{x^6}{y^6 z^2}$

Absolute Value Function:

↳ Special PW function

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$



$g_1(x) = x$

$g_2(x) = -x$

Finding Roots:

Def^n:

A root of a function $f(x)$ is a number $x_0 \in D_f$ so that

$$f(x_0) = 0$$

Ex: Find roots:

(a) $f(x) = \dfrac{3x-1}{x^2-4}$

D_f:

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

(b) $g(x) = \dfrac{x+1}{x^2+3x+2}$

D_g:

$(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

(c) $h(x) = \begin{cases} x^2 - 1, & \text{if} \quad x \le 0 \\ x+3, & \text{if} \quad 0 < x \le 3 \\ 2-x, & \text{if} \quad x > 3 \end{cases}$

(a) $\dfrac{3x-1}{x^2-4} = 0 \implies 3x - 1 = 0$

$x = 1/3$

(b) $\dfrac{x+1}{x^2+3x+2} = 0 \implies x+1 = 0$

$x = -1$

possible root!

To find domain: set denominator $= 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0 \longrightarrow$

$x = -2$

$x = -1$

bad pts

So, $x=-2$ and $x=-1$ are **NOT** in domain!

Therefore, $g(x)$ has no roots!

(c)

$$h(x) = \begin{cases} x^2-1, & \text{if } x \le 0 \\ x+3, & \text{if } 0 < x \le 3 \\ 2-x, & \text{if } x > 3 \end{cases}$$

- 1st piece:

$x^2-1=0$ $\quad (-\infty, 0]$

$x = \pm 1$

$x=1$ is not in $(-\infty, 0]$ so the only root is $x=-1$

- 2nd piece:

$x+3=0$ $\quad (0, 3]$

$x=-3$

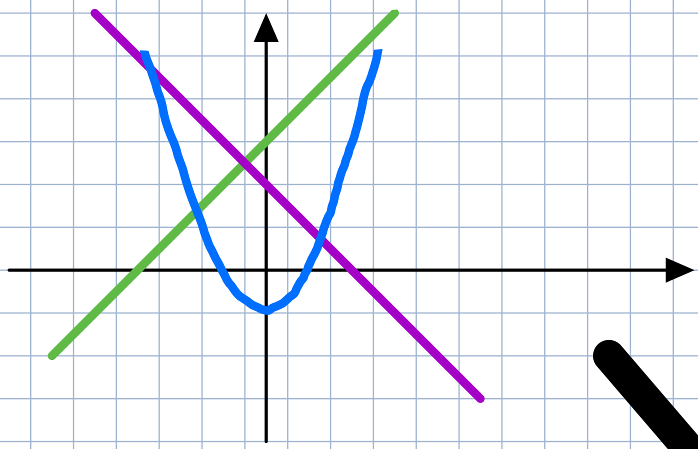
Since $x=-3$ is not in $(0,3]$, this piece has no roots!

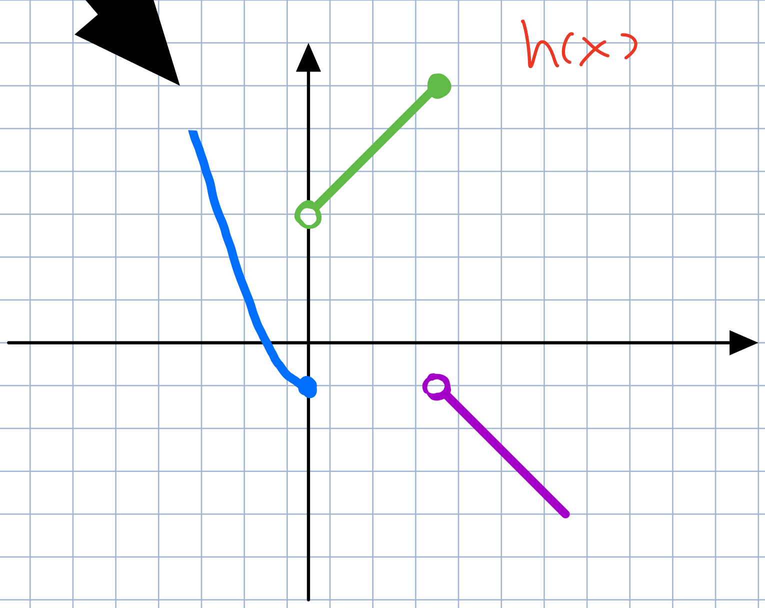
- $\underline{3^{rd} \text{ piece:}}$ $\quad 2 - x = 0 \quad (3, \infty)$

$$x = 2$$

Since $x=2$ is not in $(3,\infty)$, this piece also has no roots!

Thus, the only root for $h(x)$ is $x=-1$.





- OH HW
- HW – Sections 1 & 2 (NOT collected)

Section 3: → Commonly Encountered Functions (Poly's and Rational)

Def^n:

An n^{th} degree polynomial function has the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 x^0$$

$$n \in \{0, 1, 2, 3, \dots\}, \quad a_i\text{'s} \in \mathbb{R} \quad a_n \neq 0.$$

We're gonna focus on two simple groups:

Lines: polynomials of 1^{st} degree

All lines look like: $f(x) = a_1 x + a_0$

→ This is usually written as $y = mx + b$

"slope"

spot on y-axis where line hits i.e. "y-intercept"

Sidenote: when we write "$y = \dots$", it's understood that we have a function

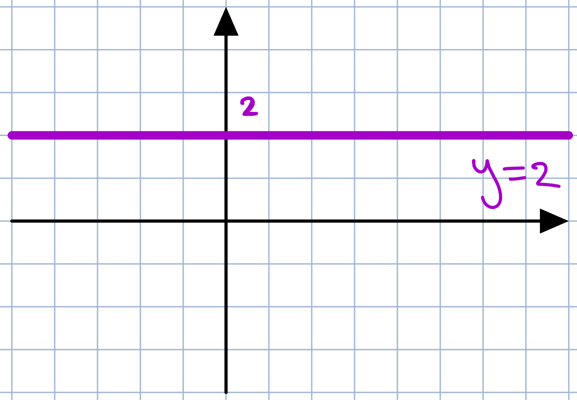
"$y(x) = \dots$"

name of function

variable / thingy ya plug into.

It turns out that this **m** determines the line's behavior!

- **If $m=0$:** $y = 0x + b \rightarrow y = b$.

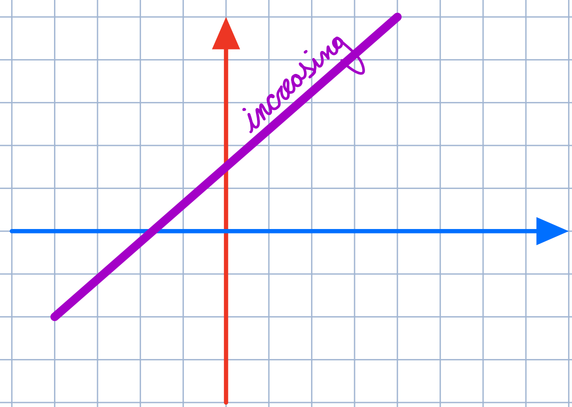


This means that the y's are all b! (no matter what x is)

example: $b=2$

- **If $m>0$:** $y = (\text{positive})\,x + b$

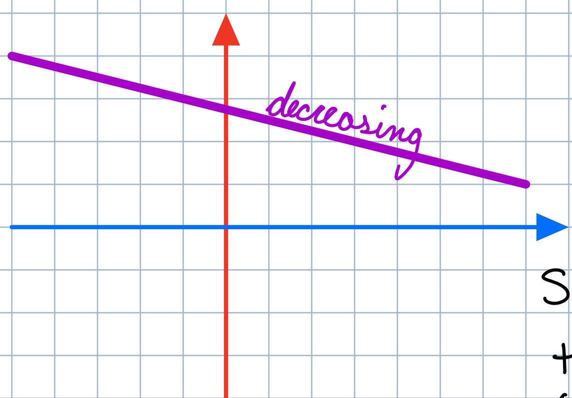
  ↳ Note: If x increases, then consequently y increases!



So moving from **Left** → **Right** (inc. in the x-direction), the function goes **up** (inc. in the y-direction)

- **If $m<0$,** $y = (\text{negative})\,x + b$

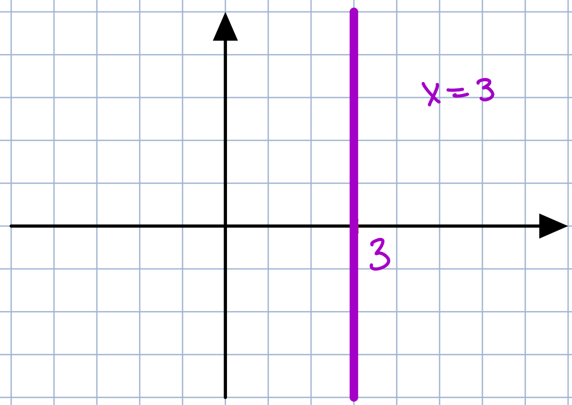
  ↳ Note: Here, if x increases, then y goes down b/c of the neg.!



So moving from **Left** → **Right** (inc. in the x-direction), the function goes **down** (dec. in the y-direction)

There is one last type of line that isn't a function...

• (m is undefined) , $x = a$

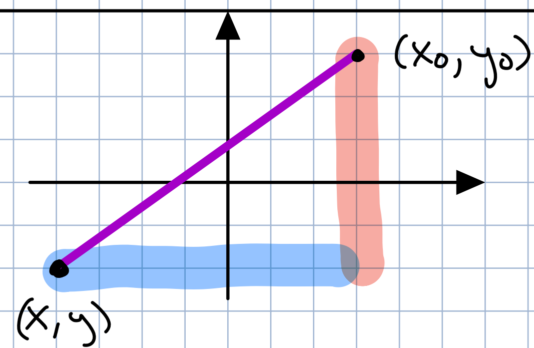


This means that the x's are all a! (no matter what y is)

These are <u>not</u> functions!

How to find slope:

$$m = \frac{\Delta y}{\Delta x}$$



$$m = \frac{y - y_0}{x - x_0}$$

Point-Slope:

$$y - y_0 = m(x - x_0)$$

(x_0, y_0) is any pt on the line

<u>Ex</u>: Find eq^n of these lines:

(a) the line that goes through $(1,1)$ & $(3,-5)$

(b) the line that goes through $(-1,2)$ with slope 4

(a) **Need:** slope, any pt!

(doesn't matter which)

$$m = \frac{\Delta y}{\Delta x} = \frac{1+5}{1-3} = \frac{6}{-2} = -3$$

Hence, our eq^n is:

$$y + 5 = -3(x - 3)$$

(b) **Need:** slope, any pt!

Hence, our eq^n is:

$$y - 2 = 4(x + 1)$$

Quadratic Functions: polynomials of 2nd degree

All quadratics look like:

$$f(x) = a_2 x^2 + a_1 x + a_0$$

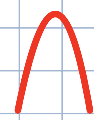
We often use the more familiar:

$$f(x) = ax^2 + bx + c.$$

Similar to the linear case, the behavior of these poly's is dominated by the highest-order term.

In this case, that's the ax^2 term!

- If $a>0$, then our parabola looks like: 

- If $a<0$, then our parabola looks like: 

How to find roots:

- Factor (guess and check)
- Factor by grouping.
- Quadratic Eq^n! (always effective)

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

will always give roots to a quadratic

How many roots should we expect?

Just look at the discriminant!

fancy word for "stuff under the radical."

$$\boxed{\Delta = b^2 - 4ac}$$

- If $\Delta < 0$, then $\sqrt{b^2-4ac}$ = imaginary!

  So, in this case, ya get no real roots!

- If $\Delta > 0$, then $\sqrt{b^2-4ac}$ = real #!

  In this case, ya get two (diff) roots!

- If $\Delta = 0$, then $\sqrt{b^2-4ac} = 0$!

  In this case, ya get 1 root (repeated)

Ex: Factor $-x^2+6x-8$.

Use quadratic eqn! $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-6 \pm \sqrt{36-4(8)}}{-2}$$

$$= \frac{-6 \pm \sqrt{36-32}}{-2} = \frac{-6 \pm \sqrt{4}}{-2} = \frac{-6 \pm 2}{-2}$$

Our two roots are: $\frac{-6+2}{-2}$ & $\frac{-6-2}{-2}$

$x = 2$ & $x = 4$

Sidenote: If $x = a$, is a root of poly $f(x)$, then $(x-a)$ is a factor of $f(x)$

Hence, $-x^2+6x-8 = (x-2)(x-4)$.

Let's try completing the square!

Goal: Rewrite quadratic as

$$a(x-k)^2 + l$$

Our quad: $-x^2+6x-8$

$$\longrightarrow -(x^2-6x+8)$$

$-(x^2-6x \quad +8)$ Use: $\frac{b}{2} \rightarrow \frac{-6}{2} = -3$

Then, square it: $\boxed{9}$

Add and subtract that value

$$-(x^2-6x+8) = -(x^2-6x+9-9+8)$$

$$= -((x-3)^2 \quad -1)$$

$$= -(x-3)^2+1$$

Ex: Complete the square: $3x^2+12x+11$

$$3x^2+12x+11 = 3(x^2+4x) + 11$$

Find $\frac{b}{2} = \frac{4}{2} = 2$. Square it: $\boxed{4}$

$$3(x^2+4x)+11 = 3(x^2+4x+4-4)+11$$

$$= 3\left[(x+2)^2-4\right]+11$$

$$= 3(x+2)^2-12+11$$

$$= 3(x+2)^2-1$$ ✓